A COMPUTER WAR GAME FOR DETERMINING THE COST EFFECTIVENESS OF ACTIVE SONOBUOYS

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A COMPUTER WAR GAME FOR DETERMINING THE COST-EFFECTIVENESS OF ACTIVE SONOBUOYS

by

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ABSTRACT

In order to evaluate the relative effectiveness of different active non-directional sonobuoys, a computer war game is developed. One submarine, employing one evasion tactic, is opposed by one helicopter, using five prosecution tactics. The tactic of the helicopter prior to the initial detection of the submarine is seen to be critical, and this simulation aids in determining an optimum tactic.

A cost-effectiveness model to use data from this simulation is developed.

An example, using hypothetical but realistic data, is presented to illustrate methods of determining the cost-effectiveness of each sonobuoy type when used with its optimum tactic.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

CHAPTER I

INTRODUCTION

The United States Navy currently uses many types of devices in the search for, and the localization and destruction of, enemy submarines.

The sonobuoy, which is an air-dropped, expendable, sonar capable of relaying its findings to the aircraft, is one such device. A particular type is the active non-directional sonobuoy, which sends a pulse of sound into the water, and transmits the sound of that pulse and the sound of an echo pulse from a possible submarine to the aircraft via radio. From the difference in the two times, the range of the submarine from the sonobuoy can be determined. The locus of the submarine's possible positions is a hemisphere of that radius about the sonobuoy.

In the Summer of 1966, while working with the Systems Analysis Staff of the Antisubmarine Warfare Systems Project Office, the author encountered the problem of developing a method of comparing the costs and effectiveness of several types of such active sonobuoys some of which were currently in use in the fleet and others of which were under consideration for future use.

The precursor of this model was originally developed and used to solve the effectiveness part of the problem, and has been further developed since then.

I. THE PROBLEM

A new anti submarine warfare (ASW) helicopter was being designed and to increase the effectiveness of the helicopter as an ASW fighting unit, the vehicle, its sensing equipment, and its kill device were being considered as a package. The kill device had been previously developed and was suitable for this helicopter. One sensing device had been incorporated into the helicopter. It was a magnetic anomoly detection (MAD) unit streamed behind the helicopter on a cable. When operated at a low altitude it can detect an anomoly in the earth's magnetic field caused by the presence of a steel submarine. MAD is a subsidiary sensing device, and is not adequate for localization when used by itself.

The main device was to be an active sonobuoy system consisting of sonobuoys and equipment in the helicopter to process the data radioed from the sonobuoys. This system was under study and various competing systems of sonobuoys were to be evaluated. A system consists of the sonobuoys and airborne receiving and processing equipment necessary for the effective use of the sonobuoys.

II. REVIEW OF THE LITERATURE

In determining whether or not a satisfactory model had been developed by another agency the Defense Documentation Center was consulted since it is the repository of technical publications for the Defense Department. The Naval Postgraduate School library was searched because student theses frequently deal with similar problems. The Applied Physics Laboratory of Johns Hopkins University was consulted since many ASW models have been developed there. Also checked was Navy War Game Manual and its classified supplement.

All searches indicated that no model had been developed to handle this problem.

III. ORGANIZATION OF REMAINDER OF THESIS

A description of the model, its purpose and assumptions will be presented next, then input instructions for running the computer program and a description of the output data will follow.

A cost-effectiveness example using hypothetical data will be followed by a summary and conclusions.

The Bibliography is followed by an Appendix presenting flow charts, definitions of program names, the computer program in FORTRAN 63, a statistical examination of two critical aspects of the model, and the results of an analysis of the model's logic.

CHAPTER II

TACTICS SIMULATION

Simulation was chosen because the problem was so complex that no mathematical model known to the author could handle it.

One helicopter against one submarine was used, the assumption being that this was the most likely combination to occur.

From the analytic insight obtained by devising and playing a manual war game based on the author's tactical experience in ASW, it quickly became apparent that the helicopters tactics were so important as to overwhelm all other parameters except the range of the sonobuoy.

It was decided to write a computer time-step simulation to evaluate the optimum tactic of the helicopter for each model of sonobuoy. Then the same program could be used to compare sonobuoy systems, using the optimum tactic for each. Here, tactic is defined as the laying of a pattern of sonobuoys on the water. The initial tactic is to lay a pattern of sonobuoys which is prescribed by doctrine for such an occasion. The pilot would continue it to completion unless a detection were obtained by one or more sonobuoys in the pattern. Subsequent tactics would depend upon the information received from the sonobuoys.

I. PURPOSE OF THE MODEL

The model has two functions. The first is to determine an optimum initial tactic to be used with each model of sonobuoy. The second is to determine the cost-effectiveness of each model of sonobuoy using the optimum initial tactic for that model.

II. DESCRIPTION OF SIMULATION

The submarine's initial position is generated at random in a square of size determined by the user. Its course is also generated randomly. It pursues that course at its patrol speed until three conditions have been met: (1) the light-off time of at least one sonobuoy has occurred, (2) the present time, in seconds, is a multiple of thirty, and (3) the submarine is within twice the input range of the sonobuoy from one or more sonobuoys. At that time, it changes course to the reciprocal of the average bearing to the sonobuoys which satisfy the above requirements. At the time of the first detection of a sonobuoy by the submarine, the submarine doubles its speed and stays at that speed the remainder of the play (replication).

The helicopter starts at its input position and flies at its input speed toward the first input sonobuoy coordinates. When it reaches that position it drops a sonobuoy and alters course toward the second input sonobuoy coordinates. The light-off time of the sonobuoy is 180 seconds after the time of the drop. The helicopter continues this sequence until it has dropped the input number of sonobuoys in the initial tactic or until at least one sonobuoy has achieved detection. In the first case, it then flies a square search of side .2 miles about the last sonobuoy coordinates. It has been determined by analysis that this square is the helicopter's optimum MAD entrapment tactic. In the second case it abandons the tactic, not to return to it again, and commences TAC1, TAC2, or TAC3.

The detections occur as follows. Every thirty seconds a check is made to see if the helicopter is within MAD range of the submarine.

If so, the replication ends in favor of the helicopter and a value of one is assigned as the outcome of that replication. If no detection occurs the game continues. Every thirty seconds all sonobuoys which have lighted off are checked to see if the submarine is within sonobuoy detection range. The coordinates of the sonobuoys achieving detection and their distances from the submarine, up to a total of three sonobuoys, are recorded.

In the following description the term "ring" will refer to a circle whose center is at a sonobuoy and whose radius is the detection range of the sonobuoy.

If the helicopter gets information from only one sonobuoy, it checks to see if it, the helicopter, is inside the ring. If so, it proceeds outward from the sonobuoy to the ring, drops a sonobuoy, goes to the opposite arc via the center and drops another sonobuoy. It then commences a square search about the last sonobuoy.

The phrase "drops a sonobuoy" used anywhere except in TACA means that subroutine ECON is called and, if no sonobuoy is within one-half of its detection range from the helicopter at the position at which it is ready to drop, a sonobuoy is dropped there. If none is dropped the helicopter continues its current tactic.

If the helicopter receives information from two sonobuoys simultaneously, it flies to the nearest point of intersection of the two rings, drops a sonobuoy, proceeds to the second intersection, drops another sonobuoy, then flies a square search about the last.

If it receives information from three sonobuoys simultaneously it proceeds to the point of mutual intersection of the three rings, drops a sonobuoy, and flies a square seach about that position.

While the above is happening, every thirty seconds the sonobuoys which have lighted off are being checked for information. If no sonobuoy is ~~acheiving~~ achieving detection, the current tactic continues. If one, two, or at least three sonobuoys are achieving detection, the one, two, or three ring tactic, respectively, is used. No information from any previous tactic is used in this case.

If the submarine input escape time occurs the play ends in favor of the submarine and a value of zero is assigned as the outcome of that replication.

III. ASSUMPTIONS MADE

Of the many assumptions made in adapting a computer model to reality, the following major ones are listed.

A. Helicopter

1. The single helicopter starts from its initial position with the crew knowing that there is a possible submarine within an area about datum which is an input for each game.

2. The helicopter is crewed by reasoning persons whose goal is to achieve MAD detection as quickly and using as few sonobuoys as possible.

3. The helicopter, at the end of each tactic during which no new information came in, flies a square search. When this is

done in the initial tactic and in the tactic which results from only one sonobuoy achieving detection (one-ring tactic), it is assumed to be waiting for information. This is equivalent to random MAD in a "real" helicopter. In the two-ring and three-ring tactics it is trying to entrap the submarine to achieve MAD detection.

B. Submarine

1. At the start of play the submarine is proceeding on a straight course toward an objective, submerged and at patrol speed, unaware that the search phase detection has been made.

2. Upon first detecting an active sonobuoy, the submarine skipper realizes he is being prosecuted by an aircraft dropping active sonobuoys in his vicinity. He changes his short term objective to eluding the aircraft, believing that if he manages to escape detection for a short time he will be able to escape the aircraft and resume his original objective. He therefore doubles his speed and attempts to elude all sonobuoys by taking as his course the reciprocal of the average bearing to all sonobuoys which have commenced emitting sound (have "lighted off") and which are within his detection range. Thereafter he computes his course anew each time new data comes in, but keeps his speed constant.

C. Sonobuoy

1. The sonobuoy has a "cooky cutter" probability distribution function.

2. It has a three minute light-off time.

3. All survive impact with the water and achieve ranges which are predictable.

D. MAD

1. The MAD gear has a "cooky cutter" probability distribution function.

2. The gear is unaffected by helicopter maneuvering.

3. The gear achieves ranges which are predictable.

E. Other Assumptions

1. Both the helicopter and submarine make pinpoint and instantaneous turns.

2. The water has constant and uniform properties.

3. Both the helicopter and the submarine have limitations upon the speed with which they may receive, process, and act on new data. The game was written so that new data can be acted on every thirty seconds of play.

4. Ranges from the helicopter to the submarine and from the sonobuoy transducer to the submarine are considered horizontal ranges, not slant ranges. This simplifying assumption approximates the case of a helicopter at low altitude, and the sonobuoy transducer and submarine at the same shallow depth.

CHAPTER III

INPUT INSTRUCTIONS

The input to this program is relatively simple. A maximum of 214 items is to be read in. One is the title of the data set, thirteen are various performance parameters and miscellaneous items, and there are up to 200 X and Y coordinates of sonobuoys. A maximum of 100 sonobuoys is allowed for the total used in all tactics of a single replication.

Figure 1 presents the card number, width of fields in terms of columns, and a description of each field or group of fields.

Card	Columns	Format	Description
1	1-12	2A6	Any alpha-numeric character may be used to designate data set number.
2	1-5	F5.1	VH = the speed of the helicopter in knots.
2	6-10	F5.1	VSS = the patrol speed of the submarine in knots.
2	11-15	F5.1	BRING = one-half of the side of a square, centered on datum, in which the submarine must start.
2	16-20	F5.1	XHH = the helicopter's starting x coordinate in nautical miles from datum, where (x, y) = (o, o) is datum.
2	21-25	F5.1	YHH = the helicopter's starting y coordinate in nautical miles from datum.
2	26-30	F5.1	TMAX = the ~~excape~~ escape time of the submarine in hours.
2	31-35	F5.1	RMAD = the radius of the MAD gear in nautical miles.
2	36-40	F5.1	RSB = the radius of the sonobuoy detecting capability in nautical miles.
2	41-51	010	IIR = the starting argument for the random number generator, in octal.
3	1-10	I10	NSB = the number of sonobuoys in the initial tactic.
3	11-20	I10	NREP = the number of replications desired in one game.
3	21-30	I10	KTEST: If zero, no statistical testing of random numbers will occur; If any positive integer is placed here, testing will occur.

FIGURE 1

INPUT DATA DESCRIPTION

Card	Columns	Format	Description
3	31-40	I10	NRAN = the number of random numbers to be tested.
4	1-5	F5.1	A(2) = the x coordinate at which the first sonobuoy in the initial tactic will be dropped, in nautical miles from datum.
4	6-10	F5.1	B(2) = the y coor dinate of the first sonobuoy.
4	11-80	7F5.1	x and y coordinates of sonobuoys two through eight.
5-12	1-80	8F5.1	As above for sonobuoys nine through sixteen etc., to a total of 100 sets of (x, y) coordinates if desired. The number of (x, y) coordinate sets must agree with NSB on card 3.

FIGURE 1 (CONTINUED)

CHAPTER IV

OUTPUT DESCRIPTION

There are three main and one supplementary outputs from this program.

A read-out of the input parameters, as explained in Figure 1, is illustrated in Figure 2(a). This is printed before the first replication starts.

At the end of each replication four lines are printed. The first is "outcome of this replication" and a 1.00 signifies a helicopter win, while a .00 signifies a submarine win. The other three are "number of sonobuoys used," "game hours to completion," and "number of this replication." These are illustrated in Figure 2(b).

At the end of the entire game four more lines are printed; "average probability of detection," "average number of sonobuoys used," "average game hours to completion," and "number of replications in the game". Figure 2(c) illustrates these lines.

If the random numbers generated are examined, a listing of significant statistical information is made. The lists are "interval", "number in this interval", "mean", "sample mean", and "sample variance". These are shown in Figure 3 for the value IIR = 47532352.

```
TACTICS 10

120.0   6.0   4.0   0-20.0   1.0    .2   3.0   47532352
          5           1            1        1355
    0  -4.0   4.0   0       0   4.0  -4.0           0      0
```

(a)

OUTCOME OF THIS REPLICATION	=	1.00
NUMBER OF SONOBUOYS USED	=	3
GAME HOURS TO COMPLETION	=	.27
NUMBER OF THIS REPLICATION	=	1

(b)

AVERAGE PROBABILITY OF DETECTION	=	.18
AVERAGE NUMBER OF SONOBUOYS USED	=	14.27
AVERAGE GAME HOURS TO COMPLETION	=	.75
NUMBER OF REPLICATIONS IN GAME	=	271

(c)

FIGURE 2

MAIN TYPES OF OUTPUT FROM PROGRAM: (a) INPUT DATA; (b) DATA FROM ONE REPLICATION AND; (c) AVERAGE DATA FOR GAME

INTERVAL	NO. IN THIS INTERVAL	MEAN	SAMPLE MEAN	VARIANCE	SAMPLE VARIANCE
0 TO .1	131	.05000	.04820	.00083	.00077
.1 TO .2	149	.15000	.14976	.00083	.00090
.2 TO .3	118	.25000	.25600	.00083	.00093
.3 TO .4	156	.35000	.34592	.00083	.00081
.4 TO .5	142	.45000	.45364	.00083	.00093
.5 TO .6	137	.55000	.55215	.00083	.00083
.6 TO .7	134	.65000	.65538	.00083	.00092
.7 TO .8	139	.75000	.75179	.00083	.00078
.8 TO .9	114	.85000	.85027	.00083	.00085
.9 TO 1.0	135	.95000	.94836	.00083	.00074

FIGURE 3

A SUPPLEMENTARY OUTPUT FOR ANALYSIS OF RANDOM NUMBERS

CHAPTER V

COST-EFFECTIVENESS EXAMPLE

This simulation model can be used to determine an optimum initial tactic for use with a given model sonobuoy system. Another use is to determine the cost-effectiveness of different active non-directional sonobuoy systems by first finding the optimum initial tactic for each, then comparing the outputs "average probability of detection" and "average number of sonobuoys used" together with cost information in a cost-effectiveness model to aid in deciding among the competing sonobuoy systems.

All sonobuoys and their performance data, all output data, and all costs are hypothetical but realistic.

I THE ASSUMED PROBLEM

One active sonobuoy, the SQS-AA, is in current use in the fleet. A new helicopter is being developed and the main sensing device for it could be one of two sonobuoys, the SQS-XX and the SQS-YY, currently in the research phase. Only theoretical performance data is available for the latter two, while for the former both theoretical performance, developed in the research phase, and observed performance data, recorded in use, are available.

For the SQS-AA two cost studies are available: The initial study for the proposed new sonobuoy SQS-AA, done in 1957; a follow up study on the sonobuoy after it had been in the fleet one year, done in 1962.

Only one cost study is available for each of the SQS-XX and SQS-YY sonobuoys; the initial studies, done in 1967.

II RESULT DESIRED

The solution to the problem will aid in determining which of the proposed new sonobuoys, if either, should enter into the development phase.

III USE OF SIMULATION MODEL

The theoretical performance data for the SQS-AA has been used because the methods used to determine it correspond to the methods used to determine performance data for the SQS-XX and SQS-YY.

The following scenario was selected. An unidentified submarine is detected by a shore unit and is determined to be within an area approximated by a square of certain input dimensions. That area is near a convoy which has a destroyer based helicopter in the air to serve as a "pouncer" aircraft. The helicopter is vectored to the area and starts a localization process as described in chapter II, using tactics which are systematically varied in a series of games.

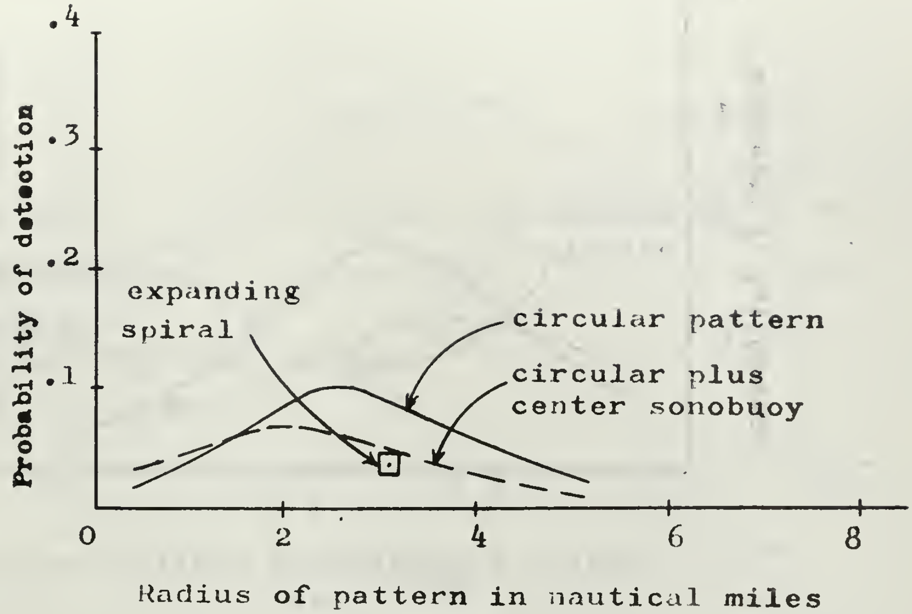
First one sonobuoy, the SQS-AA, is evaluated. Its performance data is put into the program together with a likely initial tactic which, as previously, stated, is a set of sonobuoy coordinates. The other input parameters are introduced now and will remain the same for all replications of all games of the series.

The first replication commences and the computer generates the submarine starting position within the square, and its course at random. Then the computer "moves" the helicopter and submarine until either "wins" a play. That terminates one replication of one game. For the next replication only the submarine's starting coordinates and course are different. After a preset, input, number of replications, various important data is averaged and presented on the output sheet. Then the next set of input data is introduced to start a new game. This might differ from the first set, for example, only in the radius of the pattern of sonobuoys in the initial tactic.

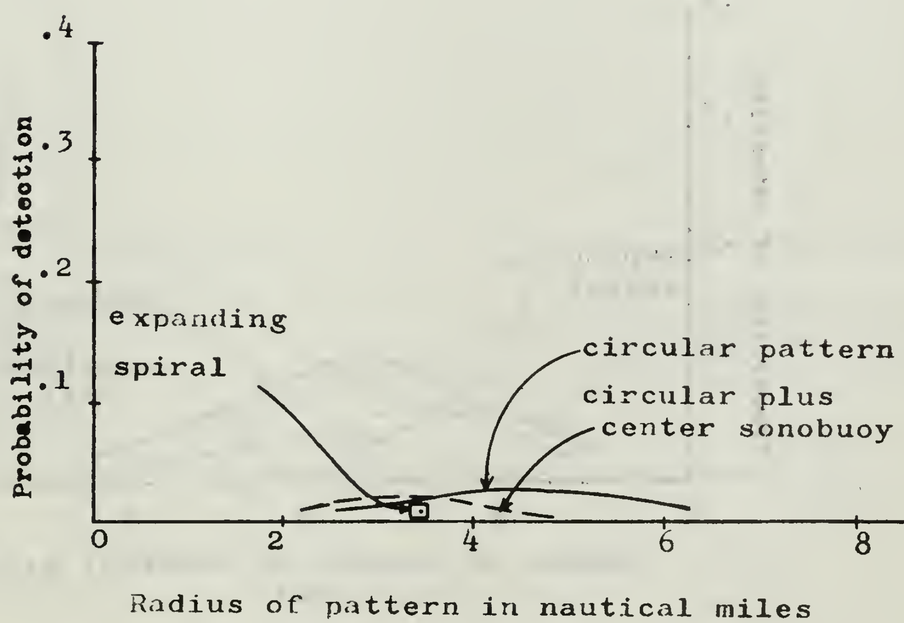
This process is continued until all sonobuoys, used with all tactics likely to bring success to the helicopter, have been investigated. The best tactic for each is determined by graphing (average) probability of detection against any other significant parameter desired. For example, in figure 4 the radius of the sonobuoy pattern about datum was chosen. Figure 4(a) shows those two tactics plotted for all tactics which utilized the SOS-AA against conventional submarines of six knots patrol speed and twelve knot "escape" speed. Figure 4(b) shows the same for nuclear submarines of fifteen knot patrol speed and thirty knot escape speed.

Figures 5 and 6 show the same type of graphs for the SQS-XX and SQS-YY respectively. The circular pattern of sonobuoys yields the highest probability of detection of the three types of tactics considered. Since the maximum ordinate on the graph falls at significantly different radii for conventional and nuclear submarines, the study would indicate different tactics against the two types of submarines. Both would employ the circular pattern but at different radii. Consequently, in the cost-effectiveness model to follow, the probability of detection for conventional and nuclear submarines will be dealth with separately. Note: These graphs are hypothetical but are similar to classified production runs made with this model.

This ends the effectiveness portion of the model and could be a separate tactics study of itself. However the optimum tactic,



(a)



(b)

FIGURE 4

PROBABILITY OF DETECTION OF (a) CONVENTIONAL AND (b) NUCLEAR SUBMARINES BY THE SQS-AA

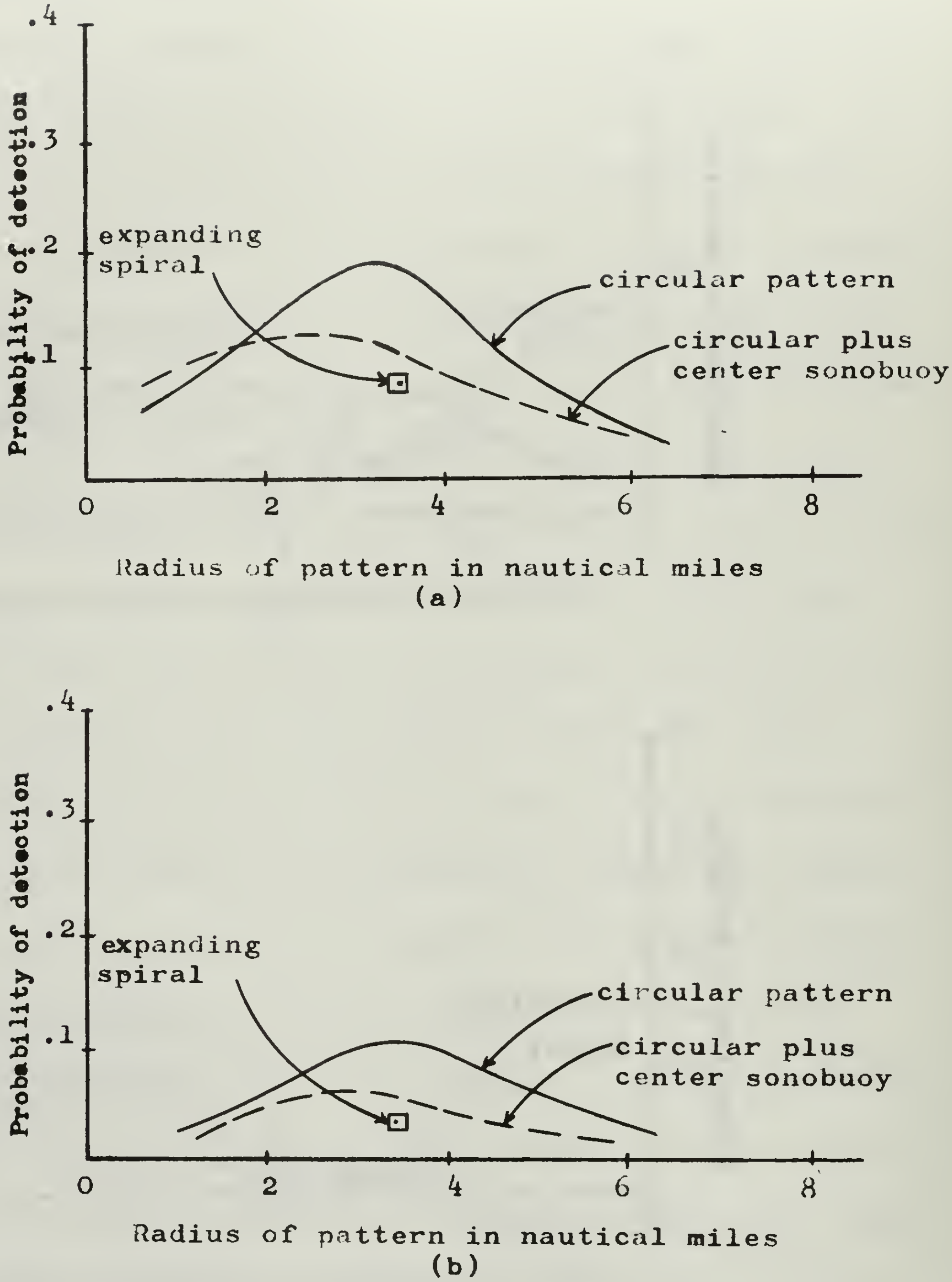


FIGURE 5

PROBABILITY OF DETECTION OF (a) CONVENTIONAL AND (b) NUCLEAR SUBMARINES BY THE SQS-XX

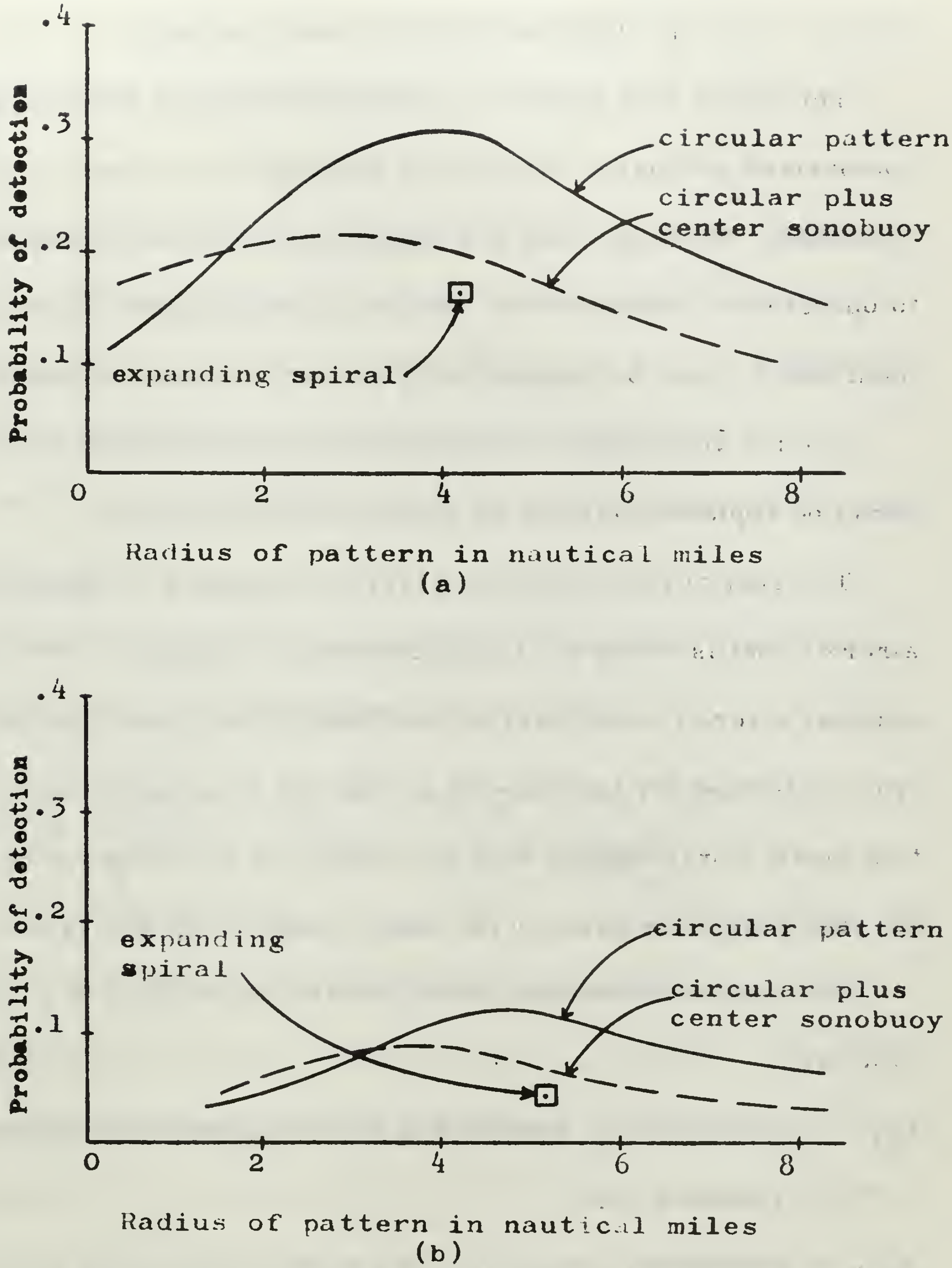


FIGURE 6

PROBABILITY OF DETECTION OF (a) CONVENTIONAL AND (b) NUCLEAR SUBMARINES BY THE SQS-YY probability of detection, and average number of sonobuoys used provide data for a cost-effectiveness study.

IV COST-EFFECTIVENESS MODEL

The initial cost studies on all three sonobuoys were selected for comparison purposes. Each study employs the ten year system cost technique. Build-up costs are negligible, since the forces which are to operate the equipment are already operating other sensing equipment which would be supplanted by the new sonobuoy systems.

The ten year figure is arrived at by estimating that the sensor would be replaced entirely by another within ten years.

The cost of the aircraft to carry the sonobuoy is ignored since the aircraft cost is added in at a higher level of analysis. Note that the original aircraft which carried the SQS-AA was fixed wing while the type envisioned for the SQS-XX or SQS-YY is rotary winged. However, this model works equally well for fixed wing and rotary wing aircraft, the only difference being in the input speeds of the two types.

The cost-effectiveness model and the terms used in it are as follows.

P_{1i} = Probability of detection of a conventional submarine by $i^{\underline{th}}$ sonobuoy type.

P_{2i} = Probability of detection of a nuclear submarine by the $i^{\underline{th}}$ sonobuoy type.

i = 1 refers to the SQS-AA

i = 2 refers to the SQS-XX

i = 3 refers to the SQS-YY

A_1 = That fraction of the enemy's subsurface fleet devoted to conventional submarines in the 1970 decade.

A_2 = That fraction of the enemy's subsurface fleet devoted to nuclear submarines in the 1970 decade.

$$(A_1 + A_2 = 1)$$

S_{1i} = The number of sonobuoys of the $i^{\underline{th}}$ type used by the helicopter in its optimum tactic against conventional submarines.

S_{2i} = The number of sonobuoys of the $i^{\underline{th}}$ type used by the helicopter in its optimum tactic against nuclear submarines.

C_i = Cost per unit of the $i^{\underline{th}}$ sonobuoy type as tabulated in column (8) of figure 7.

$(C/E)_i$ = Cost per unit of effectiveness of the $i^{\underline{th}}$ sonobuoy type where effectiveness is defined later in this section.

$$(C/E)_i = \left[(A_i S_{1i} + A_2 S_{2i}) \Big/ \frac{A_1 P_{1i} + A_2 P_{2i}}{A_1 P_{11} + A_2 P_{21}} \right] \cdot C_i$$

$(i = 1,\ 2,\ 3,)$

V RESULTS OF EXAMPLE PROBLEM

The breakdown of the costs for each model is shown in Figure 7, where all numbers represent millions of dollars. Column (1), Research and Development, is self explanatory. Column (2), Initial Investment, represents the cost of tooling up and producing the estimated number of sonobuoys needed for ten years use, listed in column (7). Annual Operations, Column (3), is the average cost of storing and maintaining the dwindling supply of sonobuoys for one year, while column (4) represents those costs for the ten year study period. Column (5) is the sum

Type	(1) Research and Development	(2) Initial Investment	(3) Annual Operations	(4) Annual Operations Times Ten
SQS-AA	$ .1	$ 1.8	$ .08	$ .8
SQS-XX	.3	7.8	.21	2.1
SQS-YY	.4	9.4	.32	3.2

Type	(5) Cost of Ten Year System	(6) Cost in 1967 Dollars	(7) Number of Sonobuoys to be Produced and Used.	(8) Cost per Sonobuoy
SQS-AA	$ 2.7	$ 3.6	40,000	$ 90
SQS-XX	10.2	10.2	60,000	170
SQS-YY	13.0	13.0	60,000	217

FIGURE 7

COST OF THREE SONOBUOY TYPES
(IN MILLIONS OF DOLLARS)

of columns (1), (2), and (4). It represents the ten year system cost in 1957 dollars for the SQS-AA, and 1967 dollars for the SQS-XX and SQS-YY. In column (6) all costs are in 1967 dollars, where the annual inflation between 1957 and 1967 was determined to be 3%. Column (8) is the ratio of the entries in column (6) to the entries in column (7), and represents the cost of one sonobuoy for each of the three models.

Figure 8 tabulates the numbers needed for the cost-effectiveness model presented in section IV of this chapter, Columns (1), (2), and (3) are taken from Figures 4, 5, and 6, and represent the Probability of Detection for nuclear and conventional submarines when the optimum tactics are used.

In the decade 1970-1980, during which this system would be employed, it is estimated that the enemy will have a mix of six conventional submarines for every four nuclear. So $A_1 = .6$ and $A_2 = .4$. Column (3) then, is the weighted mean of the items in columns (1) and (2). Columns (4) and (5) are the Average Number of Sonobuoys needed for the tactic chosen, and column (6) shows the weighted mean of the numbers in the previous two columns.

In column (7) the SQS-AA weighted mean probability of detection from column (3) is defined to be one unit of effectiveness and the effectiveness of the other two sonobuoys are computed in relation to it by dividing the entries in column (3) by .074 to obtain column (7). In column (8), the ratios of the respective entries in columns (6) and (7) are listed, and represent the number of sonobuoys needed for one unit of effectiveness.

i	Type	(1) P_{1i}	(2) P_{2i}	(3) $A_iP_{1i}+A_2P_{2i}$	(4) S_{1i}	(5) S_{2i}
1	SQS-AA	.11	.02	.074	10	18.
2	SQS-XX	.18	.10	.148	19	13
3	SQS-YY	.30	.12	.228	7	9

i	Type	(6) $A_1S_{1i}+A_2S_{2i}$ $=x$	(7) $\frac{A_1P_{1i}+A_2P_{2i}}{A_1P_{11}+A_2P_{21}}$ $=y$	(8) $\frac{x}{y}$	(9) Ci	(10) (C/E)i
1	SQS-AA	13.2	1.	13.2	$ 90	$1190
2	SQS-XX	11.2	2.00	5.60	170	952
3	SQS-YY	7.8	3.08	2.53	217	549

FIGURE 8

DATA FOR THE COST-EFFECTIVENESS MODEL

Column (9) lists the cost per unit of each type of sonobuoy as listed in column (8) of Figure 7. Column (10) is the product of the items in columns (8) and (9) and is the desired result.

Using the criteria "Minimize Cost Per Unit Effectiveness", the SQS-YY would emerge as the "best buy" despite its higher cost. This is in consonance with intuition because its range of detection as input to the program was approximately four times that of the SQS-AA, and twice that of the SQS-XX.

CHAPTER VI

SUMMARY AND CONCLUSIONS

I SUMMARY

The problem was to find a means of comparing active sonobuoys of different performance and cost, and find a way to determine the relative cost-effectiveness of each type. From playing manual war games in which one helicopter was pitted against one submarine, it became apparent that the initial tactics employed by the helicopter were of paramount importance in determining the effectiveness of each type sonobuoy, and yet the "optimal" tactic for each was not known.

A computer war game utilizing Monte Carlo technique was written when it was discovered that there was no model available in the literature to test tactics for active sonobuoys.

The model is documented in chapters II through IV, and is illustrated by means of a complete cost-effectiveness example in chapter V.

II CONCLUSIONS

The computer war game developed is capable of doing two main tasks.

It will evaluate the tactics of any aircraft using active sonobuoys so as to arrive at one or more useful tactics for each type sonobuoy studied.

Once the optimum tactic is decided upon, the numbers of sonobuoys used for each type may be used, together with cost information, to compare the sonobuoy types for cost-effectiveness.

The model has been thoroughly analysed by the statistical and geometrical methods presented in the Appendices. In addition ten hours of production runs yielded preliminary data which approximately agreed with the manual war games and coincided both with intuition and experience. That data is classified secret and has not been included in this unclassified report, which concentrates on the underlying technique.

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APPENDIX A

GENERAL FLOW CHARTS

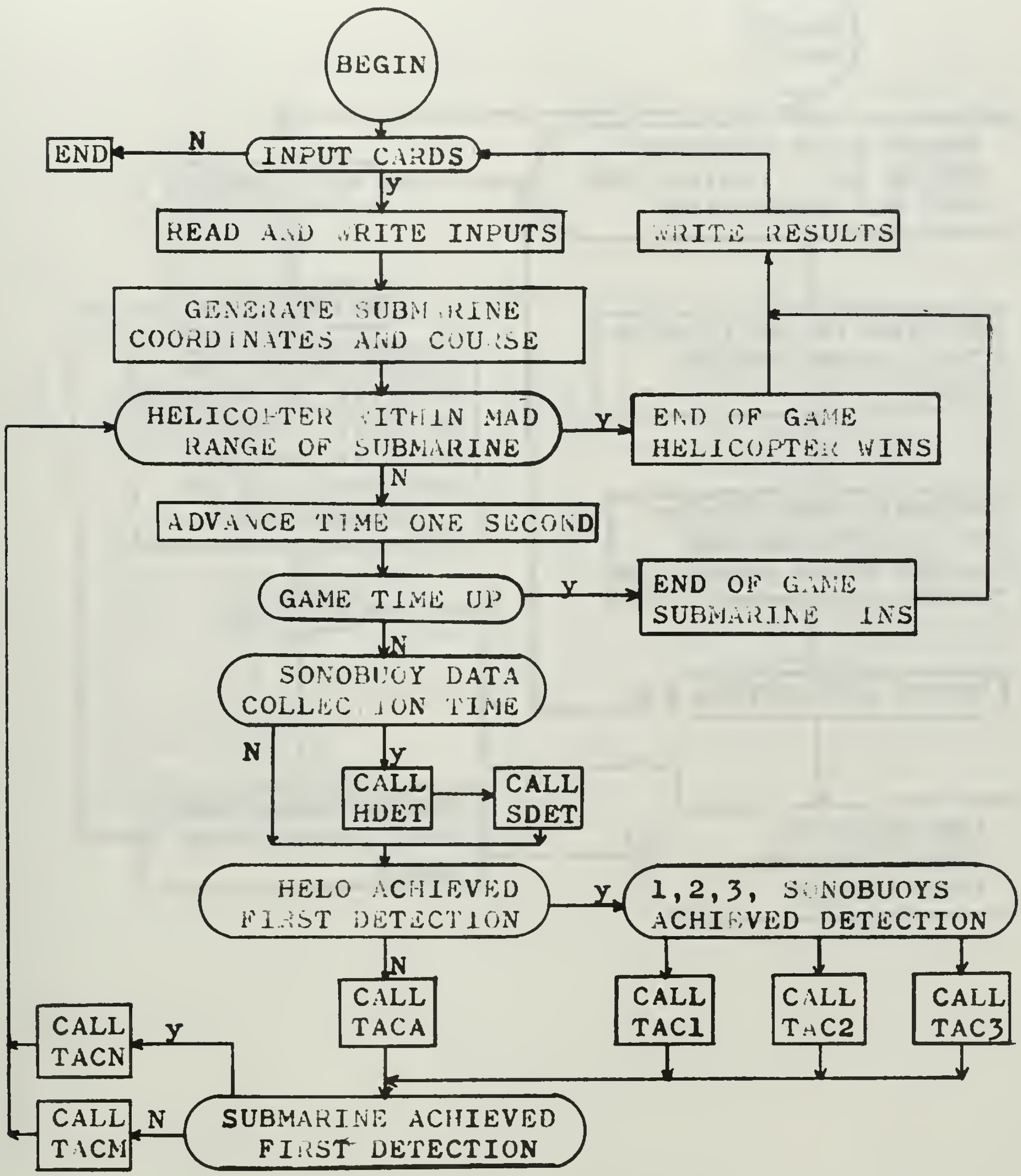


FIGURE 9

MAIN PROGRAM

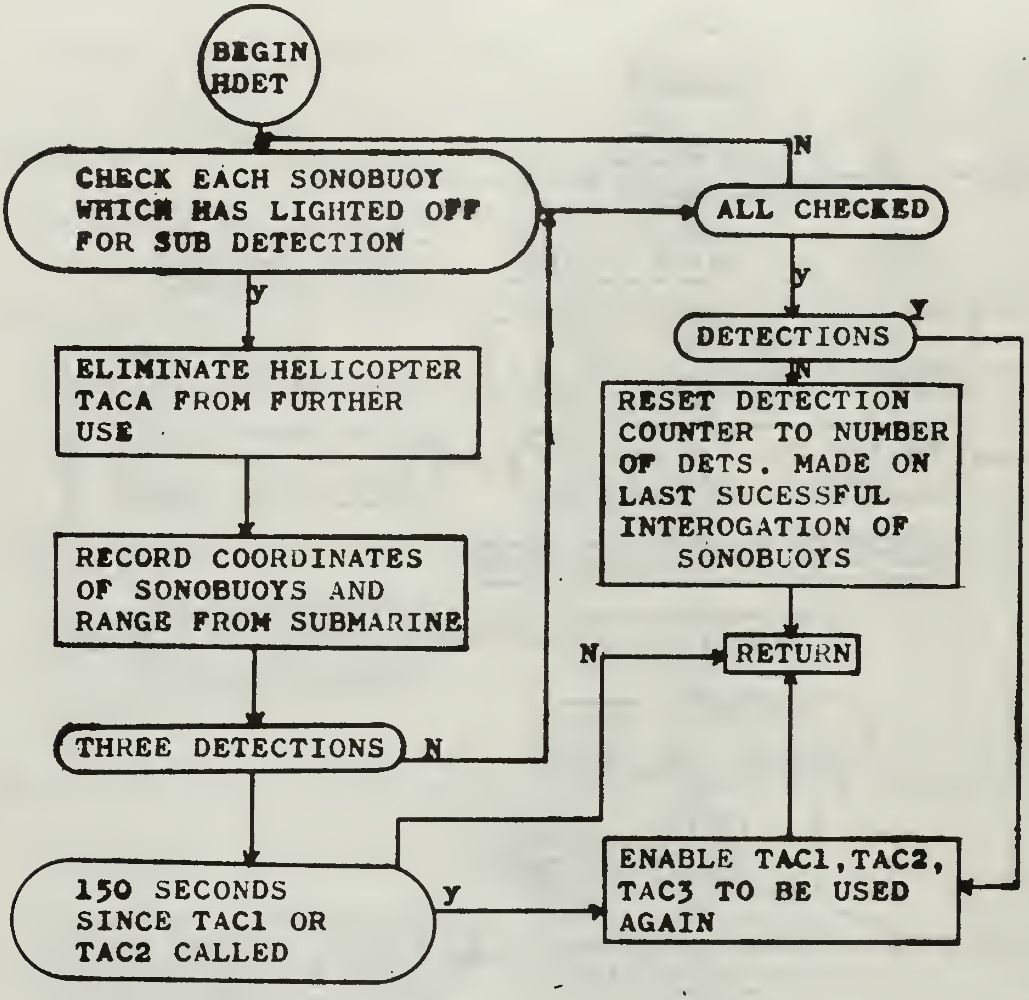


FIGURE 10

HELICOPTER-SONOBUOY DETECTION SUBROUTINE

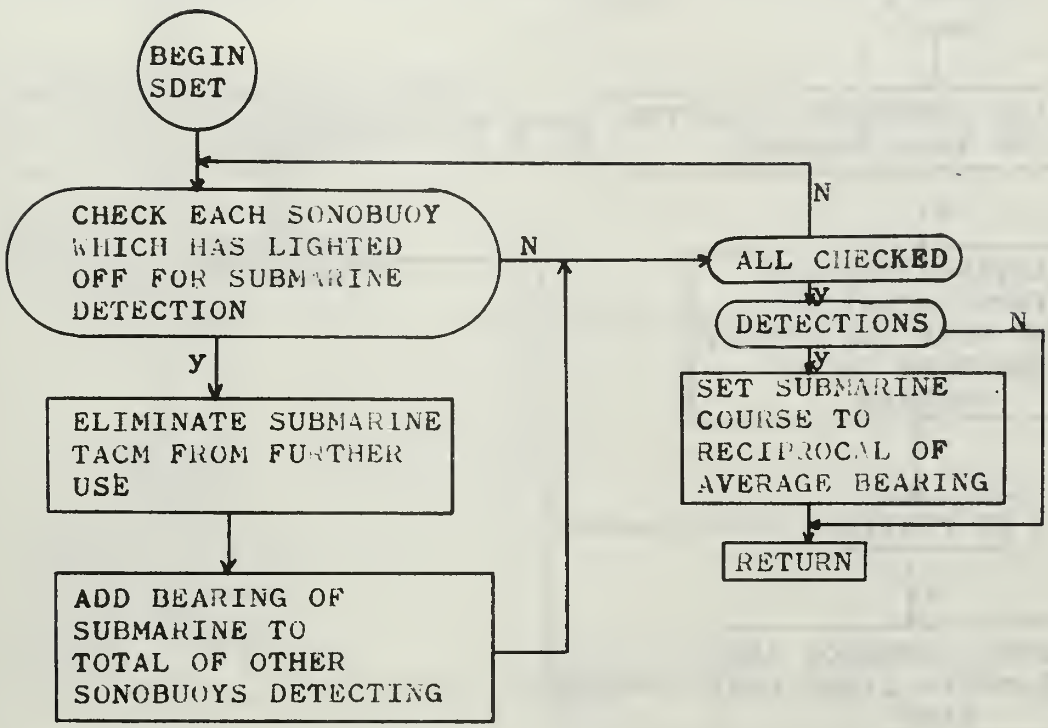


FIGURE 11

SUBMARINE DETECTION SUBROUTINE

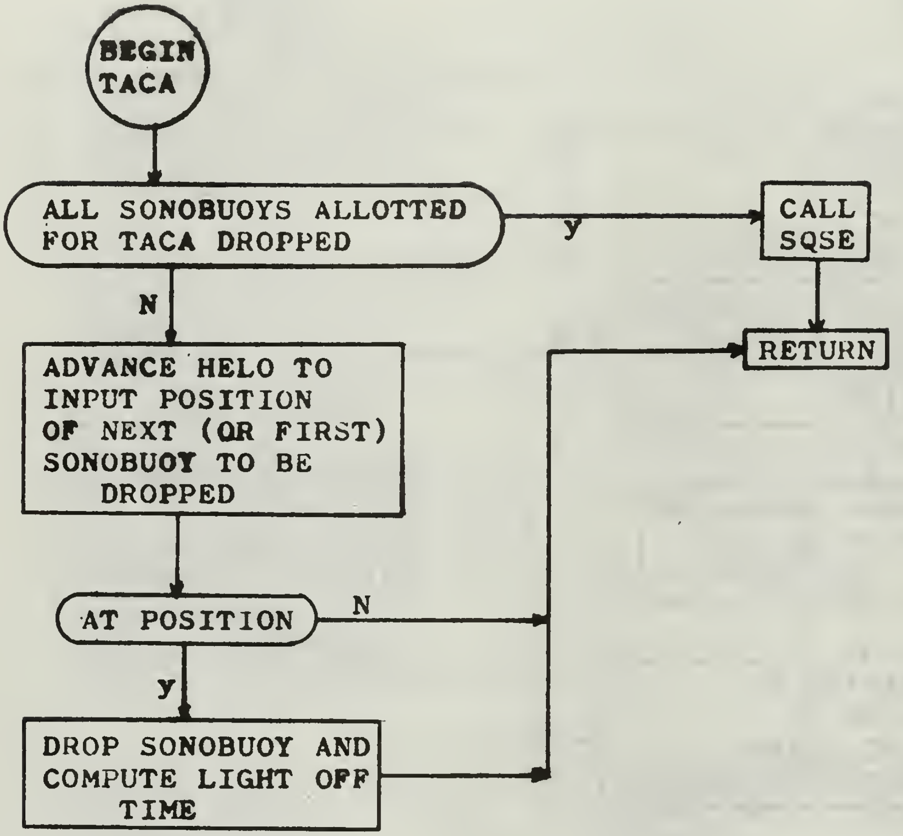


FIGURE 12

HELICOPTER INITIAL TACTIC SUBROUTINE

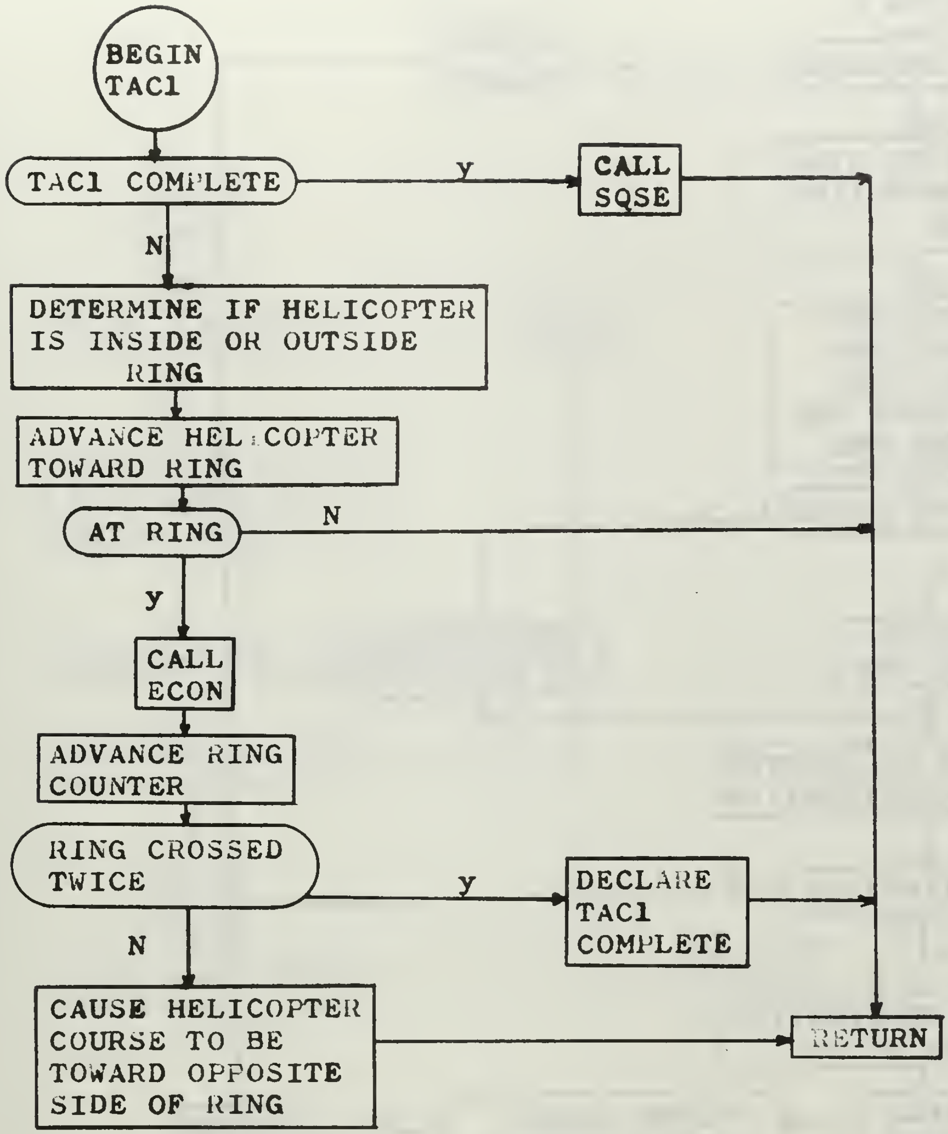


FIGURE 13

HELICOPTER TACTIC ONE-RING SUBROUTINE

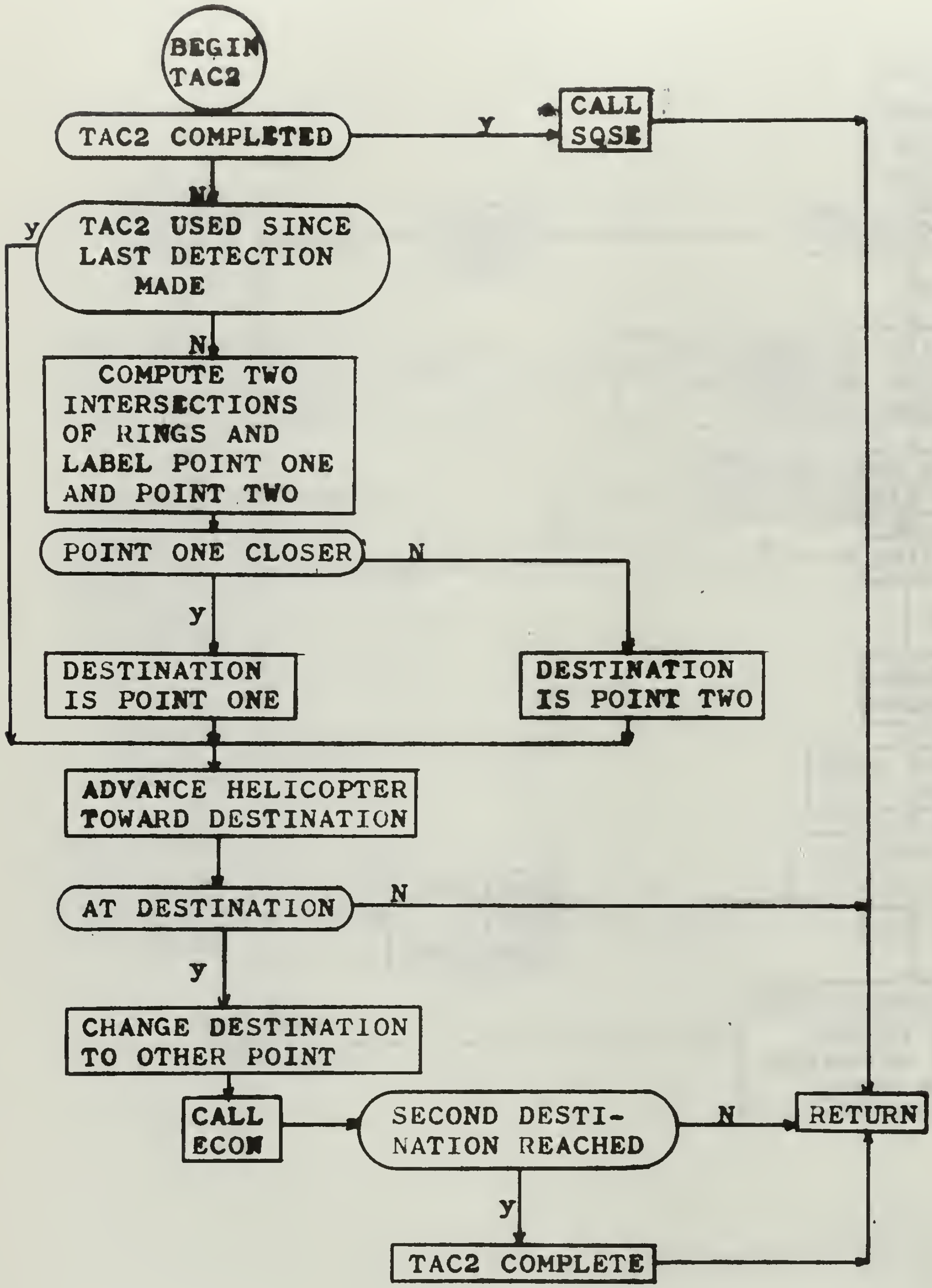


FIGURE 14

HELICOPTER TACTIC TWO-RING

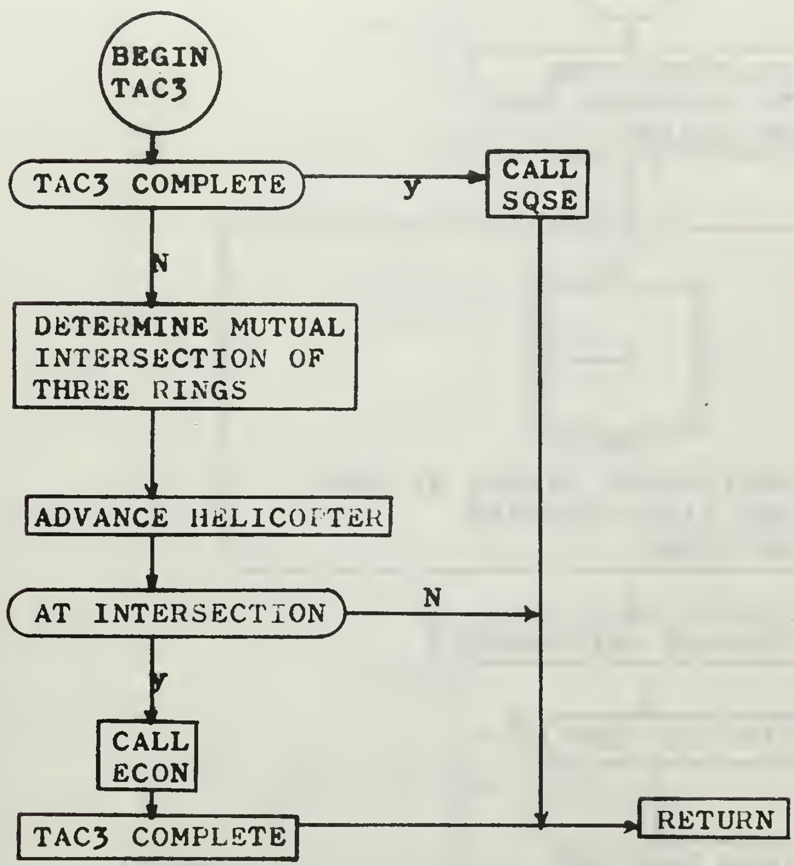


FIGURE 15

HELICOPTER TACTIC THREE-RING SUBROUTINE

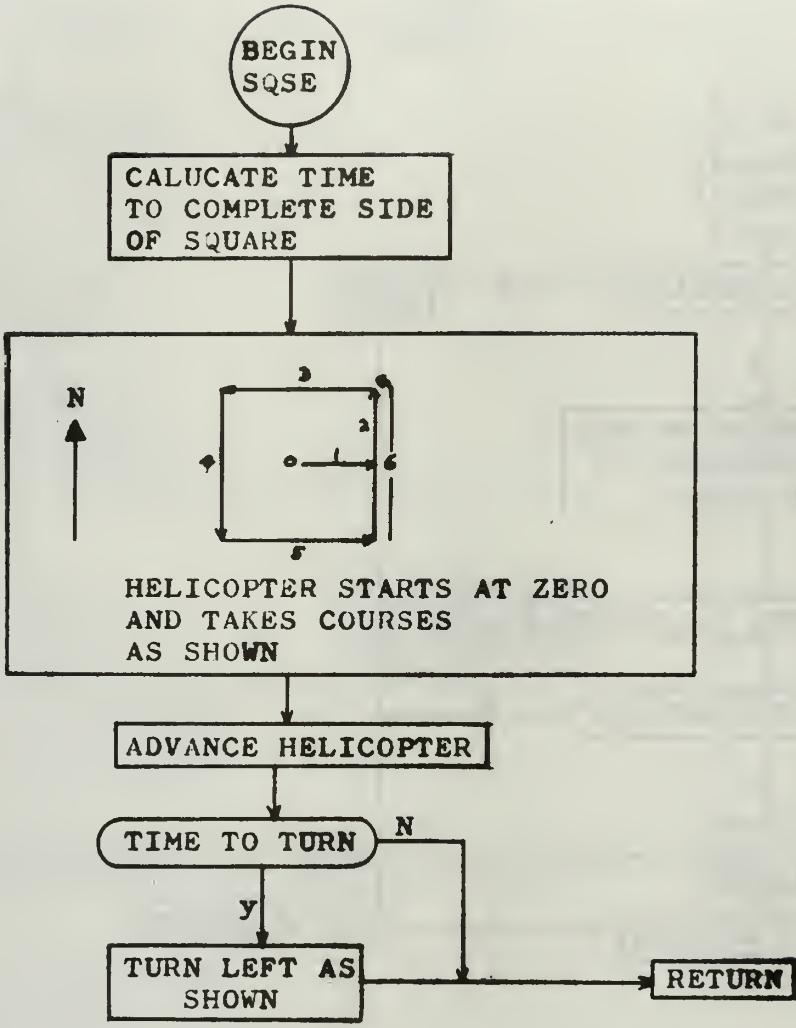
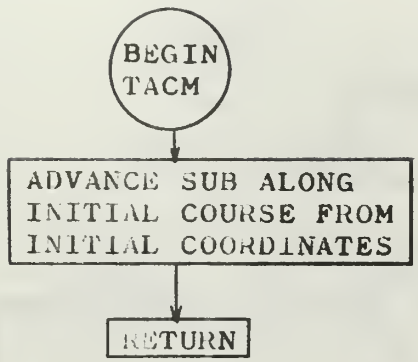
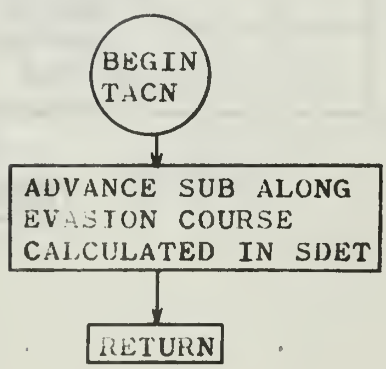


FIGURE 16

HELICOPTER SQUARE SEARCH SUBROUTINE



(a)



(b)

FIGURE 17

(a) SUBMARINE FIRST TACTIC SUBROUTINE

(b) SUBMARINE SECOND TACTIC SUBROUTINE

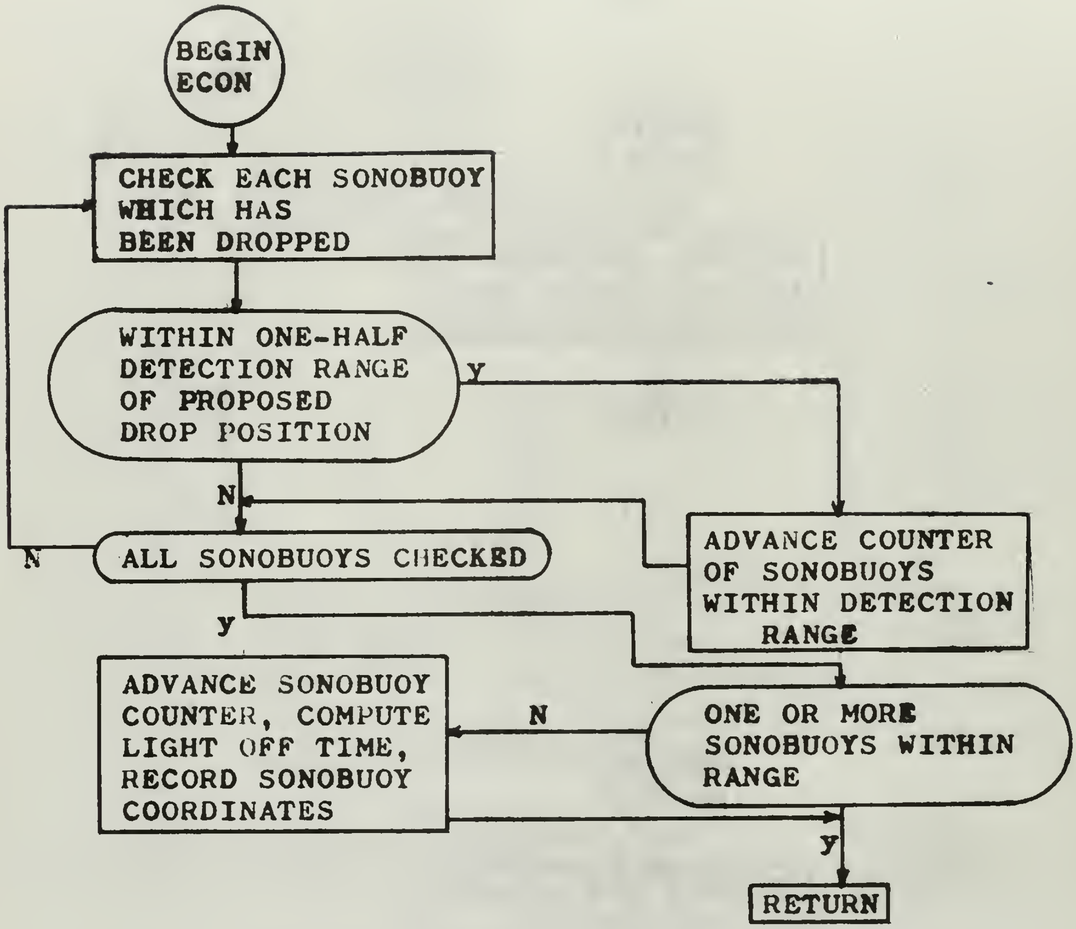


FIGURE 18

SONOBUOY ECONOMIZING SUBROUTINE

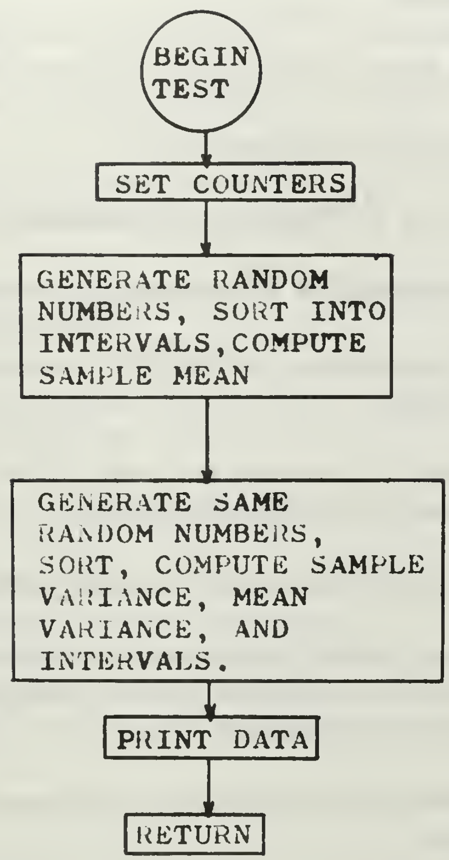


FIGURE 19

RANDOM NUMBER TEST SUBROUTINE

APPENDIX B.

DICTIONARY OF PROGRAM NAMES

AA, BB	No meaning attached; For reading in title of data set.
A(I)	Input X coordinate for $(I-1)^{th}$ sonobuoy of helicopter's initial tactic.
AK	Floating point equivalent to counter K in subroutine TEST.
ALFA	Initial course of Submarine.
AMAN(I)	Mean of the I^{th} interval in Subroutine TEST
BE(I)	Lower endpoint of I^{th} interval in subroutine TEST.
B(I)	Input Y coordinate for $(I-1)^{th}$ sonobuoy of helicopter's initial tactic.
BRING	Input value of one half the side of the square in which submarine is contained initially.
CUS	Computes escape course of Submarine in Subroutine TACN.
ECON	Subroutine which drops new sonobuoys in TAC1, TAC2, and TAC3 only ~~is~~ if there are not one or more sonobuoys within detection range.
HDET	Subroutine which queries sonobuoys for detection of submarine.
IIR	Input argument for random number subroutine. Entered in octal.
IR	Equivalent to IIR.
JA	Counter for submarine and helicopter data collection.
KCK	Prevents KTLIT(I) from being calculated more than once.
KMEAN	Tells if sample mean has been computed. Done on first pass through random numbers.
KSB	The number of sonobuoys (plus one) which have been dropped at any given time. $1 \leq KSB \leq 101$

KSBCK	Counter which prevents helicopter from dropping a sonobuoy if there are one or more within one-half of the detection range of the intended drop position.
KTEST	Input which determines whether or not subroutine TEST is called.
KTIME	Game time in seconds. $0 \leq KTIME \leq KTMAX$
KTLIT(I)	Light off time of (I-1)th sonobuoy.
KTMAX	Fixed point equivalent of $TMAX \cdot 3600$.
LL	Prevents TAC1 and TAC2 from being started over unless 150 seconds have elapsed.
M	Number of sonobuoys achieving detection at a given interogation time.
MCK	Prevents helicopter from using TACA after a detection has occured.
MS	Determines what side of square search helicopter is on.
MT	Counter of seconds of helicopter advancement along a side of the square search.
MTA	Number of seconds helicopter advances for one-half the side of the square search.
MU	Determines if TAC1 has been completed.
MX	Records value of M at start of subroutine HDET and assigns its value to M again if no detections occur.
NCK	Prevents TACM from being used after submarine first detects a sonobuoy.
NN	Determines secondary tactic within TAC1.
NP	Distinguishes between two arms of ambiguity in TAC2.
NR	Counter for number of replications
NRAN	Input which determines how many random numbers are evaluated. 1355 is the most it is necessary to call.
NREP	Maximum number of replications desired. Maximum needed is 271.

NS	Determines if TAC3 has been completed.
NSB	Number of sonobuoys in a full TACA pattern.
NT	Tells helicopter what to do within TAC2.
PROB	Number assigned when TMAX or MAD occurs.
PSUM	Numerator of average probability of detection fraction.
R	Random number in FORTRAN IV
RANDOM	FORTRAN 63 equivalent of R.
RAN1	Random number subroutine.
RDET	Radius in nautical miles from each sonobuoy achieving detection to the submarines.
RMAD	"Cooky Cutter" range of MAD gear in nautical miles.
RSB	"Cooky Cutter" range of sonobuoys, in nautical miles.
SDET	Submarine's subroutine for detection of sonobuoys.
SMEAN	Sample mean of an interval of width one-tenth in subroutine TEST.
SQSE	Subroutine which moves helicopter in a square pattern to attempt MAD detection.
SUM	Numerator of average course fraction.
SVAR(I)	Sample variance of each interval in subroutine TEST.
TACA	Helicopter's initial tactic.
TAC1	Helicopter's one-ring tactic.
TAC2	Helicopter's two-ring tactic.
TAC3	Helicopter's three-ring tactic.
TACM	Submarine's initial tactic.
TACN	Submarine's evasion tactic.
TEST	Subroutine which tests random numbers for randomness.

TMAX	Submarine's escape time in hours.
TPROB	Average probability of detection of submarine by helicopter.
U(E)	Upper endpoint of $I^{\underline{th}}$ interval in subroutine TEST.
VAR(K)	Term needed to compute sample variance.
VH	Helicopter airspeed in knots.
VSS	Submarine patrol speed in knots.
XDET(M)	X coordinate of $M^{\underline{th}}$ sonobuoy detecting submarine, in nautical miles.
XH	X coordinate of helicopter in nautical miles.
XHH	X coordinate of helicopter initial position in nautical miles.
XI	Denominator of CUS.
X(I)	X coordinate in nautical miles of $(I-1)^{\underline{th}}$ sonobuoy which has been dropped. Also serves as a counter in subroutine TEST.
XKSB	Floating point equivalent of KSB.
XNR	Floating point equivalent of NR.
XP	X coordinate in nautical miles of one point of intersection of two circles. (XP=XP1 or XP=XP2).
XP1	X coordinate of one point of intersection of two circles in TAC2 and TAC3.
XP2	X coordinate in nautical miles of the other intersection. (See XP1)
XS	X coordinate of submarine in nautical miles.
XTIME	Floating point equivalent of KTIME.
YDET(M)	Y coordinate in nautical miles of $M^{\underline{th}}$ sonobuoy detecting submarine.
YH	Y coordinate in nautical miles of helicopter.

YHH	Y coordinate in nautical miles of helicopter's starting position.
Y(I)	Y coordinate in nautical miles of $(I-1)^{\underline{th}}$ sonobuoy which has been dropped. Also serves as a counter in subroutine TEST.
YKSB	Numerator of average number of sonobuoys used.
YP	Y coordinate in nautical miles of one point of intersection of two circles. (YP=YP1 or YP=YP2).
YP1	Y coordinate in nautical miles of one point of intersection of two circles in TAC2 and TAC3.
YP2	Y coordinate in nautical miles of the other intersection. (See YP1).
YS	Y coordinate, in nautical miles, of submarine.
YTIME	Numerator of average time fraction.
Z	Term needed to compute sample variance in subroutine TEST.

Note: This program was originally written in FORTRAN IV, then altered to FORTRAN 63. Several terms, such as EQUIVALENCE (R, RANDOM), and FUNCTION SQRT, were inserted to avoid much rewriting of complicated expressions.

APPENDIX C

COMPUTER PROGRAM IN FORTRAN 63

```
      COMMON KSB,KTIME,VH,VSS, BRING,XH,YH,XS,YS,NT,MCK,CUS,ALFA,RMAD,M,
     1RSB,NSB,NS,MU,AA,BB,KCK,MS,MT,NN,NP,NCK,R,IR,RANDOM,LL,
     2X(100) ,Y(100),KTLIT(100),B(101),RDET(3),XDET(3),YDET(3),A(101)
     3,VAR(10),SMEAN(10) ,SVAR(10) ,AMAN(10) ,BE(10) ,UE(10) ,NRAN,IIR
      EQUIVALENCE(R,RANDOM)
 1001 READ(5,113)AA,BB
      READ(5,101) VH,VSS,BRING,XHH,YHH,TMAX,RMAD,RSB,IIR
      READ(5,102) NSB,NREP,KTEST,NRAN
      NSB=NSB+1
      READ(5,103)  (A(I),B(I) ,I=2,NSB)
      WRITE(6,113)AA,BB
      WRITE(6,101)VH,VSS,BRING,XHH,YHH,TMAX,RMAD,RSB,IIR
      NSB=NSB-1
      WRITE(6,102)NSB,NREP,KTEST,NRAN
      NSB=NSB+1
      WRITE(6,103) (A(I),B(I) ,I=2,NSB)
  101 FORMAT(8F5.1,O10)
  102 FORMAT(4I10)
  103 FORMAT(16F5.1)
  113 FORMAT(2A6)
      NR=0
      IR=IIR
      PSUM=0.
      YKSB=.000001
      YTIME=.000001
  109 NR=NR+1
      A(1)=XHH
      B(1)=YHH
      XNR=NR
      XH=XHH
```

```
      YH=YHH
      KCK=1
      SUM=0.
      MCK=0
      NT=1
      NN=1
      LL=6
      NS=1
      NCK=1
      NP=1
      MU=0
      MT=0
      MS=1
      M=1
      KSB=1
      KTIME=0
      JA=30
      CALL RAN1 (IR,RANDOM)
      ALFA=6.28*R
      CALL RAN1 (IR,RANDOM)
      XS=BRING*R
      CALL RAN1 (IR,RANDOM)
      YS=BRING*R
      CALL RAN1 (IR,RANDOM)
      IF(R-.5)1101,1102,1102
 1102 XS= XS
 1101 CALL RAN1 (IR,RANDOM)
      IF(R-.5)1103,140,140
 1103 YS= YS
  140 IF(SQRT((XH-XS)**2+(YH-YS)**2)-RMAD)8,8,9
    8 PROB=1.
      GO TO 100
  700 PROB=0.000001
      GO TO 100
    9 KTIME=KTIME+1
      KTMAX=TMAX*3600.
```

```
      IF(KTIME-KTMAX) 701,700,700
  701 IF(KTIME-JA)124,12,124
   12 JA=JA+30
      CALL HDET
      CALL SDET
      GO TO 124
  130 GO TO(131,132),NCK
  131 CALL TACM
      GO TO 140
  132 CALL TACN
      GO TO 140
  124 IF(MCK)125,120,125
  125 GO TO(121,122,123),M
  121 CALL TAC1
      GO TO 130
  122 CALL TAC2
      GO TO 130
  123 CALL TAC3
      GO TO 130
  120 CALL TACA
      GO TO 130
  100 PSUM=PSUM+PROB
      TPROB=PSUM/XNR
      KSB = KSB - 1
      XKSB=KSB
      YKSB=YKSB+XKSB
      XTIME=KTIME
      YTIME=YTIME+XTIME
      X=KTIME
      TIME=X/3600.
      WRITE(6,110)PROB,KSB,TIME,NR
  110 FORMAT(/,30X,30HOUTCOME OF THIS REPLICATION = F4.2,
     1      /,30X,30HNUMBER OF SONOBUOYS USED     = I3,
     2      /,30X,30HGAME HOURS TO COMPLETION     = F4.2,
     3      /,30X,30HNUMBER OF THIS REPLICATION   = I3,//)
      IF(NREP-NR) 107,107,109
```

```
107 AKSB=YKSB/(XNR+.000001)
    ATIME=YTIME/((XNR+.000001)*3600.)
    WRITE(6,342)TPROB,AKSB,ATIME,NR
342 FORMAT(//,30X,35HAVERAGE PROBABILITY OF DETECTION = F4.2,
   1       //,30X,35HAVERAGE NUMBER OF SONOBUOYS USED = F6.2,
   2       //,30X,35HAVERAGE GAME HOURS TO COMPLETION = F4.2,
   3       //,30X,35HNUMBER OF REPLICATIONS IN GAME    = I3,/////)
    IF(KTEST-1)1001,941,941
941 CALL TEST
    CONTINUE
    GO TO 1001
    END
    SUBROUTINE HDET
    COMMON KSB,KTIME,VH,VSS, BRING,XH,YH,XS,YS,NT,MCK,CUS,ALFA,RMAD,M,
   1RSB,NSB,NS,MU,AA,BB,KCK,MS,MT,NN,NP,NCK,R,IR,RANDOM,LL,
   2X(100) ,Y(100),KTLIT(100),B(101),RDET(3),XDET(3),YDET(3),A(101)
   3,VAR(10),SMEAN(10) ,SVAR(10) ,AMAN(10) ,BE(10) ,UE(10) ,NRAN,IIR
    EQUIVALENCE(R,RANDOM)
    I=1
    MX=M
    M=0
 20 I=I+1
    IF(KSB-I)928,749,749
749 IF(KTIME-KTLIT(I))928,22,22
 22 IF(SQRT((X(I)-XS )**2+(Y(I)-YS )**2)-RSB)23,23,24
 23 M=M+1
    MCK=1
    RDET(M)=SQRT((X(I)-XS )**2+(Y(I)-YS )**2)
    XDET(M)=X(I)
    YDET(M)=Y(I)
    IF(M-3)24,422,422
 24 IF(I-101)20,928,928
928 IF(M)422,421,422
421 M=MX
    GO TO 640
422 NS=1
```

```
      LL=LL-1
      IF(LL)13,13,640
   13 NN=1
      NT=1
      MU=0
      NP=1
      MS=1
      MT=0
      LL=5
  640 RETURN
      END
      SUBROUTINE  SDET
      COMMON KSB,KTIME,VH,VSS, BRING,XH,YH,XS,YS,NT,MCK,CUS,ALFA,RMAD,M,
     1RSB,NSB,NS,MU,AA,BB,KCK,MS,MT,NN,NP,NCK,R,IR,RANDOM,LL,
     2X(100) ,Y(100),KTLIT(100),B(101),RDET(3),XDET(3),YDET(3),A(101)
     3,VAR(10),SMEAN(10) ,SVAR(10) ,AMAN(10) ,BE(10) ,UE(10) ,NRAN,IIR
      EQUIVALENCE(R,RANDOM)
      SUM=.0
      ZI=0.
      I=2
      IF(KSB-2)19,484,664
  484 IF(KTIME-KTLIT(I))19,18,18
  664 I=I+1
      IF(KSB-I)224,492,492
  492 IF(KTIME-KTLIT(I))224,664,664
  224 I=I-1
      IF(I-1)19,19,18
   18 DO 14 J=2,I
      IF(SQRT((X(J)-XS)**2+(Y(J)-YS)**2) 2.*RSB)643,643,14
  643 NCK=2
      ZI=ZI+1.
      IF((Y(J)-YS)/(X(J)-XS))13,13,17
   13 IF(Y(J)-YS)11,11,16
   17 IF(Y(J)-YS)15,15,12
   16 SUM=SUM-ATANF(ABSF((Y(J)-YS)/(X(J) XS)))
      GO TO 14
```

```
15 SUM=SUM+ATANF(ABSF((Y(J)-YS)/(X(J) XS)))
   GO TO 14
12 SUM=SUM+ATANF(ABSF((Y(J)-YS)/(X(J) XS)))+3.1416
   GO TO 14
11 SUM=SUM-ATANF(ABSF((Y(J)-YS)/(X(J) XS)))+3.1416
14 CONTINUE
   CUS=SUM/ZI
19 RETURN
   END
   SUBROUTINE TACA
   COMMON KSB,KTIME,VH,VSS, BRING,XH,YH,XS,YS,NT,MCK,CUS,ALFA,RMAD,M,
  1RSB,NSB,NS,MU,AA,BB,KCK,MS,MT,NN,NP,NCK,R,IR,RANDOM,LL,
  2X(100) ,Y(100),KTLIT(100),B(101),RDET(3),XDET(3),YDET(3),A(101)
  3,VAR(10),SMEAN(10) ,SVAR(10) ,AMAN(10) ,BE(10) ,UE(10) ,NRAN,IIR
   EQUIVALENCE(R,RANDOM)
   IF(KSB-NSB)4,6,6
10 KSB=KSB+1
   X(KSB)=XH
   Y(KSB)=YH
   KTLIT(KSB)=KTIME+180
 5 RETURN
 4 I=KSB
   ZZ1=A(I+1)-A(I)
   ZZ2=B(I+1)-B(I)
   VHH=VH/3600.
   TI=SQRT(ZZ1**2+ZZ2**2)/VHH
   DX=ZZ1/TI
   DY=ZZ2/TI
   XH=XH+DX
   YH=YH+DY
   ZZ4=A(I+1)-XH
   ZZ5=B(I+1)-YH
   IF(SQRT(ZZ4**2+ZZ5**2)-2.*SQRT(DX**2+DY**2))10,10,5
 6 CALL      SQSE
   GO TO 5
   END
```

```
      SUBROUTINE TAC1
      COMMON KSB,KTIME,VH,VSS, BRING,XH,YH,XS,YS,NT,MCK,CUS,ALFA,RMAD,M,
     1RSB,NSB,NS,MU,AA,BB,KCK,MS,MT,NN,NP,NCK,R,IR,RANDOM,LL,
     2X(100) ,Y(100),KTLIT(100),B(101),RDET(3),XDET(3),YDET(3),A(101)
     3,VAR(10),SMEAN(10) ,SVAR(10) ,AMAN(10) ,BE(10) ,UE(10) ,NRAN,IIR
      EQUIVALENCE(R,RANDOM)
   52 GO TO(50,51,155),NN
   50 IF(SQRT((XDET(1)-XH)**2+(YDET(1)-YH)**2)-RDET(1))53,54,55
   55 NN=2
      GO TO 52
   53 SX=    ((YDET(1)-YH)/SQRT((YDET(1)-YH)**2+(XDET(1)-XH)**2))
      CX=    ((XDET(1)-XH)/SQRT((YDET(1)-YH)**2+(XDET(1)-XH)**2))
      XH=XH-VH*CX/3600.
      YH=YH-VH*SX/3600.
      IF(SQRT((XDET(1)-XH)**2+(YDET(1)-YH)**2)-RDET(1))56,54,54
   56 RETURN
   61 XH=XH+DX
      YH=YH+DY
      NN=1
      GO TO 56
   54 MU=MU+1
      IF(MU-2)98,98,59
   98 CALL ECON
      NN=2
      GO TO 56
   59 NN=3
      GO TO 56
  155 CALL SQSE
      GO TO 56
   51 SY=    ((YDET(1)-YH)/SQRT((YDET(1)-YH)**2+(XDET(1)-XH)**2))
      CY=    ((XDET(1)-XH)/SQRT((YDET(1)-YH)**2+(XDET(1)-XH)**2))
      DY=VH*SY/3600.
      DX=VH*CY/3600.
      XH=XH+DX
      YH=YH+DY
      IF(SQRT((XDET(1)-XH)**2+(YDET(1)-YH)**2)-SQRT(DX**2+DY**2))61,61,1
```

```
  1 50
150 IF(MU-1)160,56,59
160 IF(SQRT((XDET(1)-XH)**2+(YDET(1)-YH)**2)-RDET(1))54,54,56
    END
    SUBROUTINE TAC2
    COMMON KSB,KTIME,VH,VSS, BRING,XH,YH,XS,YS,NT,MCK,CUS,ALFA,RMAD,M,
   1RSB,NSB,NS,MU,AA,BB,KCK,MS,MT,NN,NP,NCK,R,IR,RANDOM,LL,
   2X(100) ,Y(100),KTLIT(100),B(101),RDET(3),XDET(3),YDET(3),A(101)
   3,VAR(10),SMEAN(10) ,SVAR(10) ,AMAN(10) ,BE(10) ,UE(10) ,NRAN,IIR
    EQUIVALENCE(R,RANDOM)
    GO TO(29,35,38),NT
 29 D=SQRT((XDET(1)-XDET(2))**2+(YDET(1)-YDET(2))**2)
    S=.5*(RDET(1)+RDET(2)+D)
    CA=(D**2+RDET(1)**2-RDET(2)**2)/(2.*D*RDET(1))
    SA=2.*SQRT(S*(S-RDET(1))*(S-RDET(2))*(S-D))/(D*RDET(1))
    CB=(D**2+RDET(2)**2-RDET(1)**2)/(2.*RDET(2))
    SB=(2.*SQRT(S*(S-RDET(1))*(S-RDET(2))*(S-D)))/(D*RDET(2))
    XP1=XDET(1)+CA*RDET(1)
    YP1=YDET(1)+SA*RDET(1)
    XP2=XDET(2)+CB*RDET(2)
    YP2=YDET(2)+SB*RDET(2)
    IF(SQRT((XH-XP1)**2+(YH-YP1)**2)-SQRT((XH-XP2)**2+(YH-YP2)**2))30,
   131,31
 30 XP=XP1
    YP=YP1
    NP=2
    GO TO 32
 31 XP=XP2
    YP=YP2
 32 CC=   ((XP-XH)/SQRT((XP-XH)**2+(YP YH)**2))
    SC=   ((YP-YH)/SQRT((XP-XH)**2+(YP YH)**2))
    DX=VH*CC/3600.
    DY=VH*SC/3600.
    XH=XH+DX
    YH=YH+DY
    IF(SQRT((XP-XH)**2+(YP-YH)**2)-SQRT(DX**2+DY**2))134,134,33
```

```
 33 RETURN
134 CALL ECON
    NT=NT+1
    GO TO 33
 35 GO TO(36,37),NP
 37 XP=XP2
    YP=YP2
    GO TO 32
 36 XP=XP1
    YP=YP1
    GO TO 32
 38 CALL SQSE
    GO TO 33
    END
    SUBROUTINE TAC3
    COMMON KSB,KTIME,VH,VSS, BRING,XH,YH,XS,YS,NT,MCK,CUS,ALFA,RMAD,M,
   1RSB,NSB,NS,MU,AA,BB,KCK,MS,MT,NN,NP,NCK,R,IR,RANDOM,LL,
   2X(100) ,Y(100),KTLIT(100),B(101),RDET(3),XDET(3),YDET(3),A(101)
   3,VAR(10),SMEAN(10) ,SVAR(10) ,AMAN(10) ,BE(10) ,UE(10) ,NRAN,IIR
    EQUIVALENCE(R,RANDOM)
    GO TO(64,66),NS
 64 D=SQRT((XDET(1)-XDET(2))**2+(YDET(1)-YDET(2))**2)
    S=.5*(RDET(1)+RDET(2)+D)
    CA=(D**2+RDET(1)**2-RDET(2)**2)/(2.*D*RDET(1))
    SA=2.*SQRT(S*(S-RDET(1))*(S-RDET(2))*(S-D))/(D*RDET(1))
    CB=(D**2+RDET(2)**2-RDET(1)**2)/(2.*RDET(2))
    SB=(2.*SQRT(S*(S-RDET(1))*(S-RDET(2))*(S-D)))/(D*RDET(2))
    XP1=XDET(1)+CA*RDET(1)
    XP2=XDET(2)+CB*RDET(2)
    YP1=YDET(1)+SA*RDET(1)
    YP2=YDET(2)+SB*RDET(2)
    IF(ABS(SQRT((XP1-XDET(3))**2+(YP1-YDET(3))**2)- RDET(3))-.00005)61
   1 ,6 ,60
 67 CA1=   ((XP-XH)/SQRT((XP-XH)**2+(YP-YH)**2))
    SA1=   ((YP-YH)/SQRT((XP-XH)**2+(YP-YH)**2))
    XH=XH+VH*CA1/3600.
```

```
      YH=YH+VH*SA1/3600.
      DX=VH*CA1/3600.
      DY=VH*SA1/3600.
      IF(SQRT((XP-XH)**2+(YP-YH)**2)-SQRT(DX**2+DY**2))163,163,65
  163 CALL ECON
      NS=2
   65 RETURN
   61 XP=XP1
      YP=YP1
      GO TO 67
   60 XP=XP2
      YP=YP2
      GO TO 67
   66 CALL SQSE
      GO TO 65
      END
      SUBROUTINE SQSE
      COMMON KSB,KTIME,VH,VSS, BRING,XH,YH,XS,YS,NT,MCK,CUS,ALFA,RMAD,M,
     1RSB,NSB,NS,MU,AA,BB,KCK,MS,MT,NN,NP,NCK,R,IR,RANDOM,LL,
     2X(100) ,Y(100),KTLIT(100),B(101),RDET(3),XDET(3),YDET(3),A(101)
     3,VAR(10),SMEAN(10) ,SVAR(10) ,AMAN(10) ,BE(10) ,UE(10) ,NRAN,IIR
      EQUIVALENCE(R,RANDOM)
      NNSQ=NNSQ+1
      MTA=500./VH
      GO TO(70,71,72,73,74,75),MS
   70 XH=XH+VH/3600.
      MT=MT+1
      IF(MT-MTA)77,78,78
   78 MS=2
      MT=0
   77 RETURN
   71 YH=YH+VH/3600.
      MT=MT+1
      IF(MT-MTA)77,79,79
   79 MS=3
      MT=0
```

```
      GO TO 77
   72 XH=XH-VH/3600.
      MT=MT+1
      IF(MT-2*MTA)77,80,80
   80 MS=4
      MT=0
      GO TO 77
   73 YH=YH-VH/3600.
      MT=MT+1
      IF(MT-2*MTA)77,81,81
   81 MS=5
      MT=0
      GO TO 77
   74 XH=XH+VH/3600.
      MT=MT+1
      IF(MT-2*MTA)77,82,82
   82 MS=6
      MT=0
      GO TO 77
   75 YH=YH+VH/3600.
      MT=MT+1
      IF(MT-2*MTA)77,83,83
   83 MS=3
      MT=0
      GO TO 77
      END
      SUBROUTINE TACM
      COMMON KSB,KTIME,VH,VSS, BRING,XH,YH,XS,YS,NT,MCK,CUS,ALFA,RMAD,M,
     1RSB,NSB,NS,MU,AA,BB,KCK,MS,MT,NN,NP,NCK,R,IR,RANDOM,LL,
     2X(100) ,Y(100),KTLIT(100),B(101),RDET(3),XDET(3),YDET(3),A(101)
     3,VAR(10),SMEAN(10) ,SVAR(10) ,AMAN(10) ,BE(10) ,UE(10) ,NRAN,IIR
      EQUIVALENCE(R,RANDOM)
      DX=VSS*COS(ALFA)/3600.
      DY=VSS*SIN(ALFA)/3600.
      XS=XS+DX
      YS=YS+DY
```

```
      RETURN
      END
      SUBROUTINE TACN
      COMMON KSB,KTIME,VH,VSS, BRING,XH,YH,XS,YS,NT,MCK,CUS,ALFA,RMAD,M,
     1RSB,NSB,NS,MU,AA,BB,KCK,MS,MT,NN,NP,NCK,R,IR,RANDOM,LL,
     2X(100) ,Y(100),KTLIT(100),B(101),RDET(3),XDET(3),YDET(3),A(101)
     3,VAR(10),SMEAN(10) ,SVAR(10) ,AMAN(10) ,BE(10) ,UE(10) ,NRAN,IIR
      EQUIVALENCE(R,RANDOM)
      DX=2.*VSS*COS(CUS)/3600.
      DY=2.*VSS*SIN(CUS)/3600.
      XS=XS+DX
      YS=YS+DY
      RETURN
      END
      SUBROUTINE ECON
      COMMON KSB,KTIME,VH,VSS, BRING,XH,YH,XS,YS,NT,MCK,CUS,ALFA,RMAD,M,
     1RSB,NSB,NS,MU,AA,BB,KCK,MS,MT,NN,NP,NCK,R,IR,RANDOM,LL,
     2X(100) ,Y(100),KTLIT(100),B(101),RDET(3),XDET(3),YDET(3),A(101)
     3,VAR(10),SMEAN(10) ,SVAR(10) ,AMAN(10) ,BE(10) ,UE(10) ,NRAN,IIR
      EQUIVALENCE(R,RANDOM)
      KSBCK=0
      DO 499 I=2,KSB
      IF (SQRT((X(I)-XH)**2+(Y(I)-YH)**2)-.5*RSB)479,479,499
  479 KSBCK = KSBCK+1
  499 CONTINUE
      IF(KSBCK-1)493,495,495
  493 KSB=KSB+1
      KTLIT(KSB)=KTIME+180
      X(KSB)=XH
      Y(KSB)=YH
  495 RETURN
      END
      SUBROUTINE TEST
      COMMON KSB,KTIME,VH,VSS, BRING,XH,YH,XS,YS,NT,MCK,CUS,ALFA,RMAD,M,
     1RSB,NSB,NS,MU,AA,BB,KCK,MS,MT,NN,NP,NCK,R,IR,RANDOM,LL,
     2X(100) ,Y(100),KTLIT(100),B(101),RDET(3),XDET(3),YDET(3),A(101)
```

```
     3,VAR(10),SMEAN(10) ,SVAR(10) ,AMAN(10) ,BE(10) ,UE(10) ,NRAN,IIR
      EQUIVALENCE(R,RANDOM)
      DO 25 J=1,10
   25 SMEAN(J)=0.
      KMEAN=0
   34 DO 37 K=1,10
      VAR(K)=0.
      X(K)=0.
   37 Y(K)=0.
      IR=IIR
      DO 74 I=1,NRAN
      CALL RAN1(IR,RANDOM)
      IF(R-.1)1,1,12
   12 IF(R-.2)2,2,13
   13 IF(R-.3)3,3,14
   14 IF(R-.4)4,4,15
   15 IF(R-.5)5,5,16
   16 IF(R-.6)6,6,17
   17 IF(R-.7)7,7,18
   18 IF(R-.8)8,8,19
   19 IF(R-.9)9,9,10
    1 Y(1)=Y(1)+1.
      X(1)=X(1)+R
      Z=(R-SMEAN(1))**2
      VAR(1)=VAR(1)+Z
      GO TO 74
    2 Y(2)=Y(2)+1.
      X(2)=X(2)+R
      Z=(R-SMEAN(2))**2
      VAR(2)=VAR(2)+Z
      GO TO 74
    3 Y(3)=Y(3)+1.
      X(3)=X(3)+R
      Z=(R-SMEAN(3))**2
      VAR(3)=VAR(3)+Z
      GO TO 74
```

```
 4 Y(4)=Y(4)+1.
   X(4)=X(4)+R
   Z=(R-SMEAN(4))**2
   VAR(4)=VAR(4)+Z
   GO TO 74
 5 Y(5)=Y(5)+1.
   X(5)=X(5)+R
   Z=(R-SMEAN(5))**2
   VAR(5)=VAR(5)+Z
   GO TO 74
 6 Y(6)=Y(6)+1.
   X(6)=X(6)+R
   Z=(R-SMEAN(6))**2
   VAR(6)=VAR(6)+Z
   GO TO 74
 7 Y(7)=Y(7)+1.
   X(7)=X(7)+R
   Z=(R-SMEAN(7))**2
   VAR(7)=VAR(7)+Z
   GO TO 74
 8 Y(8)=Y(8)+1.
   X(8)=X(8)+R
   Z=(R-SMEAN(8))**2
   VAR(8)=VAR(8)+Z
   GO TO 74
 9 Y(9)=Y(9)+1.
   X(9)=X(9)+R
   Z=(R-SMEAN(9))**2
   VAR(9)=VAR(9)+Z
   GO TO 74
10 Y(1 )=Y(10)+1.
   X(1 )=X(10)+R
   Z=(R-SMEAN(10))**2
   VAR(10)=VAR(10)+Z
74 CONTINUE
   IF(KMEAN)32,32,33
```

```
   32 DO 84 J=1,10
      SMEAN(J)=X(J)/Y(J)
   84 CONTINUE
      KMEAN=1
      GO TO 34
   33 DO 63 K=1,10
      AK=K
      SVAR(K)=VAR(K)/Y(K)
      AMAN(K)=(.2*AK-.1)/2.
      VAR(K)=.01/12.
      BE(K)=AK*.1-.1
   63 UE(K)=AK*.1
      WRITE(6,47)
      WRITE(6,48) (BE(I),UE(I),Y(I),AMAN(I),SMEAN(I),VAR(I),SVAR(I) ,I=1
     1,10)
   47 FORMAT(5X,8HINTERVAL,6X,20HNO. IN THIS INTERVAL,8X,4HMEAN,12X,11HS
     1AMPLE MEAN,11X,8HVARIANCE,8X,15HSAMPLE VARIANCE///)
   48 FORMAT(10(4X,F3.1,4H TO F3.1,12X,F4.,14X,F7.5,14X,F7.5,13X,F7.5,13
     1X,F7.5,//)//)
      RETURN
      END
          IDENT       RAN1
          ENTRY       RAN1                    (IR,RNDOM)
RAN1      SLJ         **
          SIU     1   R4
          LIU     1   RAN1
          LDA     1   0
          INI     1   1
          SIL     1   R4
          SAL         R3
          ARS         24
          SAU         R1
          SAL         R2
R1        LDA         **                      IR
          MUF         =1220703125
R2        DVF         =1B46
```

```
        STQ        **                      IR
        LLS        48
        DVF        =1B46
        ARS        11
        ADD        =1B46
R3      FAD        =1B46
        STA        **                      RNDOM
R4      ENI    1   **
        SLJ        **
        END
      FUNCTION SQRT (X)
      SQRT = SQRTF (X)
      RETURN
      END
      FUNCTION SIN (X)
      SIN = SINF(X)
      RETURN
      END
      FUNCTION COS(X)
      COS = COSF(X)
      RETURN
      END
      FUNCTION ABS (X)
      ABS = ABSF(X)
      RETURN
      END
        END
        FINIS
```

568

APPENDIX D

STATISTICAL EXAMINATION OF MODEL

In order that the user of the model may decide on the degree of confidence to place in the model's results, two critical features are examined statistically. Those examinations are presented in this appendix.

I RANDOM NUMBERS

Five random numbers are generated in each replication. They control the area of the square the submarine is contained in at the start, and the initial course of the submarine. The random number generator is assumed to be uniform between zero and one but, as is well known, the numbers from a random generator are not random at all, but are instead a rigid sequence of numbers determined by the program which generated them and the intial argument specified by the user. The user should examine those random numbers which his program uses, to determine if they are close enough to true randomness to meet the requirements of his problem. If they are not, one standard procedure to effect a change is to change the initial argument until a nearly random sequence of numbers has been obtained. That has been done for this model and the results and statistical methods follow.

As will be explained in the subsequent section of this appendix, 271 replications are the most runs necessary to achieve eighty one per cent confidence that the probabilities of detection of two games which differ by one-tenth are statistically different. 1355 random numbers are the most which will ever be needed, and sequences of that length have been examined. The input values of KTEST=1, NRAN= 1355, and

IIR=47532352 (octal field) were used and the results were as presented in Figure 3 of Chapter IV.

Subroutine TEST called 1355 numbers, sorted each into 10 intervals of one-tenth width each (arbitrary) counted the numbers in each interval and computed their sample mean as (4):

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

where n is the number of random numbers which fell into a given interval and X_i is the value of the $i^{\underline{th}}$ number in that interval.

The same random numbers were generated again and this time the sample variance was computed as (4):

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

where the symbols n, X_i, and $\bar{X}$ have the same meaning as before.

Next the mean (5),

$$E(X) = \frac{a+b}{2}$$

where a is the lower limit of the interval and b it's upper limit, was computed for each interval. The variance as determined by (5):

$$\sigma^2 = \frac{(b-a)^2}{12}$$

was then determined to be a constant in all intervals.

The upper and lower limits were then computed and all the above information presented in the following order: Interval limits; Number of random numbers which fell into this interval; Mean; Sample mean; Variance; and Sample variance.

The numbers in Figure 3 are the result. Since the number in each interval is close to 135.5 and there is close agreement between the mean and sample mean, and the variance and sample variance, these random numbers are regarded by the author as sufficient for the purposes of this model, hence the initial argument shown is recommended.

II PROBABILITY OF DETECTION

The outcome of each replication is a one if MAD occurs and a zero if it does not. The only things which change from one replication to the next are the submarine's starting coordinates and course. These are determined by the random numbers which, we have seen, are reasonably random. So the outcome of each replication, or "experiment" may be considered belonging to two categories, called "successes" or "failures" and represent independently repeated, or "Bernoulli trials" (5).

The maximum likehood estimator for the mean of the Bernoulli distribution is (4)

$$\hat{p} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

A large enough sample (271) is present for the normal approximation to be accurate. A ninety per cent confidence that $\hat{P}$ is within $\pm$ .05 of the "true" population mean is desired. The model for the large sample confidence interval of the normal approximation of the Bernoulli distribution is (4): P(90% of sample means will be between P+.05 and P-.05)

$$= Prob\left(\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq P \leq \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = .90$$

$$Prob\left(P \leq \hat{p} + 1.645\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = .95$$

$$\text{let } P = \hat{P} + .05$$

$$\hat{P} + .05 \leq \hat{P} + 1.645\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$$

$$\sqrt{\frac{\hat{P}(1-\hat{P})}{n}} \leq \frac{.05}{1.645}$$

$$\frac{\hat{P}(1-\hat{P})}{n} \leq \left(\frac{.05}{1.645}\right)^2$$

$$\frac{n}{\hat{P}(1-\hat{P})} \geq \left(\frac{1.645}{.05}\right)^2$$

$$n \geq 1080\left(\hat{P} - \hat{P}^2\right)$$

$$\left(\hat{P} - \hat{P}^2\right) \text{ is a maximum at } \hat{P} = \tfrac{1}{2}$$

$$n \geq 271 \quad \left(\text{at } \hat{P} = \tfrac{1}{2}\right)$$

So the most replications ever necessary is 271, and that only if the probability of detection is .5. If the probabilities are different, the number of replications required for the same confidence interval is less. For example, if $\hat{P} = .1$, then $n \geq 98$.

The estimator $\hat{P}$ will fall within .05 of the "true" value of P 90% of the time. But we wish to know if a probability that differs by .1

from another represents a "better" tactic. In order to resolve the two points each of which is 90% certain, each must lie within $\pm$ .05 of it's true mean. So if the points are separated by .1 we may be (.90)(.90) $\cdot$ 100 = 81% confident that the two points are "statistically different". Assuming the model does not bias one tactic over another, we may be 81% certain that the tactic with the higher probability of detection is "better" than the other. The author sees no evidence of such bias.

APPENDIX E

ANALYSIS OF MODEL'S LOGIC

A complete analysis of one replication is presented geometrically in Figure 20 and in the following text. The solid line in the Figure represents the helicopter's path and the broken line that of the submarine's. The arcs with numbers on them indicate the detection time from the sonobuoy in the center of the arc. Presented at the top of Figure 20 is a write out of the input to this game. This is a replication which the helicopter "won", and illustrates many of the model's features.

Cards were inserted into the program to cause certain critical values to print out every one second of game time. These values represent the positions of the helicopter and submarine at all times, positions of the sonobuoys, the positions and ranges to the submarine of all sonobuoys up to a total of three achieving detection at any given time, and the current game time in seconds.

TIME	EVENT
0 sec.	Helicopter starts at (x, y)=(0, -20), (not shown). Submarine starts at (2.2, .2).
478	Helicopter lays sonobuoy No. 1 at (0, -4.1).
648	Helicopter lays sonobuoy No. 2 at (4.0, -.1).
658	Sonobuoy No. 1 lights off.
660	Submarine detects sonobuoy No. 1, changes its course to go away from it, and doubles its speed.
818	Sonobuoy No. 3 is dropped.

```
120.0   6.0   4.0      0-20.0   1.0    .2   3.0   47532352
          5                8           0           0
    0  -4.0   4.0          0      0   4.0  -4.0     0      0      0
```

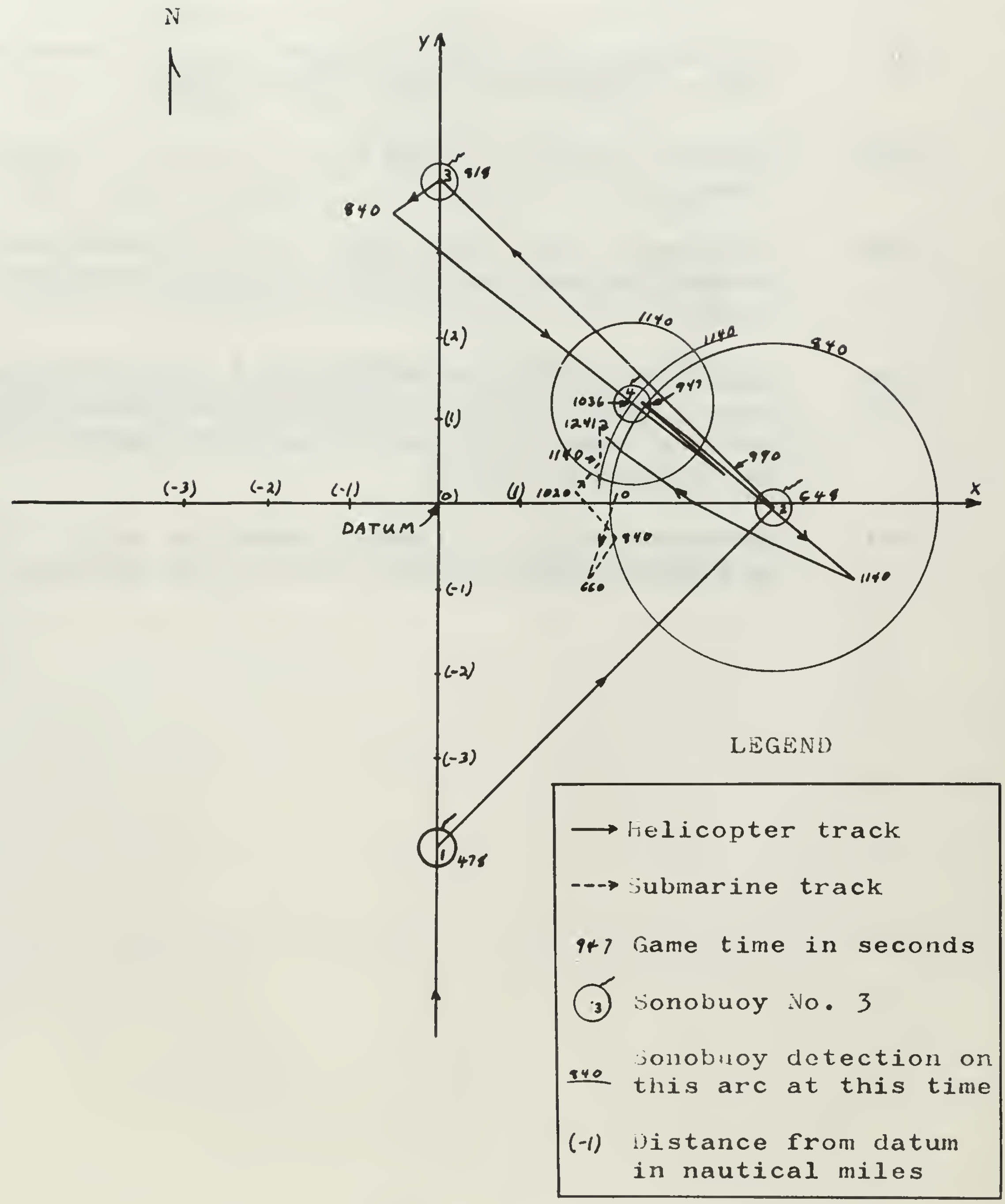


FIGURE 20

A PLAY WHICH THE HELICOPTER "WON"

TIME	EVENT
828	Sonobuoy No. 2 lights off.
840	Submarine detects sonobuoy Nos. 1 and 2 and alters course. Helicopter detects submarine on sonobuoy No. 2 and alters course as directed by TAC1.
947	Helicopter drops sonobuoy No. 4 on the ring of TAC1, at (2.3,1.2).
990	150 seconds delay time has elapsed since first detection. Helicopter reverses course to restart TAC1.
1020	Submarine detects sonobuoy Nos. 1, 2, and 3 and alters course.
1036	Helicopter reaches ring of TAC1 but is prevented from dropping because sonobuoy No. 4 is nearby. Reverses course.
1140	Submarine detects sonobuoys No. 1, 2, 3, and 4 and alters course. Helicopter detects submarine on sonobuoys No. 2 and 4 and alters course toward the nearest intersection of the two rings as in TAC2.
1241	Helicopter, at (2.0, .8) detects submarine, at (1.9, .9), by MAD and replication ends in favor of the helicopter.

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13. ABSTRACT

In order to evaluate the relative effectiveness of different active non-directional sonobuoys, a computer war game is developed. One submarine, employing one evasion tactic, is opposed by one helicopter, using five prosecution tactics. The tactic of the helicopter prior to the initial detection of the submarine is seen to be critical, and this simulation aids in determining an optimum tactic.

A cost-effectiveness model to use data from this simulation is developed.

An example, using hypothetical but realistic data, is presented to illustrate methods of determining the cost-effectiveness of each sonobuoy type when used with its optimum tactic.

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Sonobuoy						
Helicopter						
Simulation						
Monte Carlo						
Tactics						
Aircraft						
Cost-Effectiveness						
War Game						